The Pennsylvania State University The Graduate School

## COORDINATED TRANSPORT OF A SLUNG LOAD BY A TEAM OF AUTONOMOUS ROTORCRAFT

A Thesis in Aerospace Engineering by ZuQun Li

 $\bigodot$  2014 ZuQun Li

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The thesis of ZuQun Li was reviewed and approved<sup>\*</sup> by the following:

Jacob W. Langelaan Associate Professor of Aerospace Engineering Thesis Advisor

Joseph F. Horn Professor of Aerospace Engineering

George A. Lesieutre Professor of Aerospace Engineering Head of the Department of Aerospace Engineering

\*Signatures are on file in the Graduate School.

## Abstract

This research is motivated by the significant economical efficiency of multilift system for carrying a single heavy payload, which includes both military and commercial cargo.

This thesis seeks to develop a solution for the more general problem of multilft that uses two or more autonomous rotorcraft to transport a single payload. This is accomplished with Object Based Task Level Control (OBTLC), where a complex control system of multilift is separated into three control layers. Top-level is the payload trajectory following controller, which is a closed-loop controller computes net force and moment on the load's center of gravity so a desired trajectory is followed. Mid-level is the cable force control, which computes cable force at each attachment point so that the net force and moment on the CG equal to the desired value from the trajectory following controller. Finally, low-level is the flight controller onboard each helicopter that ensures the desired cable force and direction are satisfied.

Mid-level control is the main focus of this thesis. A two-step cable force computation is developed: first, solve the least-norm solution to satisfy the desired net force and moment required for payload to follow its trajectory; second, compute null space of cable force to ensure other constraints (such as vehicle separation and cable force constraint) are met. Condition number is used to select the location of attachment points so that any error in the cable force will have minimum effect on the payload. Additionally, cable force angle constraint is developed to improve stability of the payload.

A multilift simulation, where each helicopter, cable, and the payload have their own equation of motion, is used to test and validate the aforementioned approach. Result of simulations showing four degree of freedom transport of a load (North, East, Down position and yaw angle) are used to show the utility.

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# Dedication

To my loving family: Naijian, Baoqin, Ting, Lan, and Jia Juan



## Introduction

This thesis seeks to develop a solution for the more general problem of multilift that uses two or more autonomous rotorcraft to cooperatively carry a single payload. The research is motivated by the economical efficiency of using multiple rotorcraft to carry a single heavy payload, and these payload are relatively few. On the other hand, developing larger and higher lifting capacity rotorcraft is not only economically inefficient but also ineffective to manufacture huge rotorcraft those full-lifting capacity will be rarely used [2]. Therefore, a robust multilift system is a more effective way to solve the heavy lifting problem, and to fully utilize the capacity of rotorcraft.

The concept of using two helicopters to cooperatively carry a single slung load (twin-lift) has been examined several times over the past few decades for both manned and unmanned helicopters [3, 5–8]. However, a more general solution to the multilift problem must be developed to fully utilize the concept. This problem is exceptionally challenging because as the number of rotorcraft increases, overall system complexity increases and the coordinating control along typical flight paths becomes extremely difficult.

This thesis: (a) describes a solution to the general multilift problem using Object Based Task Level Control; (b) describes a cable force computation algorithm that will satisfy constraints for operational safety while providing desired net force and moment for the payload to follow the command; (c) describes how the location of attachment points on payload will affect its stability; (d) introduces cable force angle constraint for helicopter formation to improve stability of the payload.

## 1.1 Motivation

Rotorcraft are often the most effective way to transport cargo to and from areas that fixed-wing aircraft cannot reach safely. Single-helicopter slung-load operation was commonly used since 1950s, and these operations were further developed and extensively used during the Vietnam war. However, the maximum load capacity of current rotorcraft is somewhat limited. Many heavy military vehicles and cargos cannot be transported by a single rotorcraft. While larger and higher lifting capacity rotorcraft could be developed to solve this problem, it is very inefficient to develop and manufacture a huge rotorcraft whose full-lifting capacity will be rarely used. Earlier studies has show that as the rotorcraft become larger, the productivity gains with the size of rotorcraft have been leveling off. Figure 1.1 illustrates this trend [2]. Due to this fact, developing super large lifting capacity rotorcraft has its limitation.



Figure 1.1. Ideal relative productivity at 100 nautical miles [2].

With the cooperation of multiple rotorcraft, transporting heavy cargos and vehicles can be much more economical and effective. Moreover, with a solution to general multilift problem that is expandable, the heavy lifting problem can eventually be solved. Most of the research conducted to date have focused on twin-lift system including the development of equations of motion, operational concepts, and stability control [3, 5, 9–11]. This thesis is focused on the more general problem of multilift. It seeks to develop a solution that is scalable at least up to reasonable numbers of rotorcraft.

The solution is a hierarchical approach, based on a concept known as Object Based Task Level Control (OBTLC), developed for robotic manipulator system [12, 13]. In this framework top-level control is abstracted to the level of the desired payload trajectory and lower-level controllers aboard each robot determine the individual control inputs required to ensure that the desired payload trajectory is followed. In the application examined here, top level control is a trajectory following controller that computes desired payload acceleration, middle level control computes the cable forces that will result in the desired acceleration, and low level control is controller aboard each helicopter that ensures the required cable tension and cable angle are actually flown. This approach has several advantages: it is scalable, allowing the team (or flock) of rotorcraft to grow as the payload weight increases; it is possible to bring a human into the loop at several levels (for example, a human operator could "steer the payload" while the rotorcraft steer themselves in a way that best allows the payload to follow commanded inputs; a human could also take over at the level of rotorcraft control, although this could be difficult for a human pilot); finally, the abstractions at each level mean that the implementation details at each level do not have a significant effect (beyond performance constraints) on other parts of the control architecture.

## 1.2 Previous and Related Work

The use of two or more rotorcraft to carry a single payload has been investigated many times since the early success of single slung load rotorcraft operation in 1950s. These investigations were mostly sponsored by the Department of Defense. Sikorsky was funded in 1968 for studies of possible twin-lift techniques. The result of the study was a successful demonstration of a twin-lift system using two CH-54B helicopters in 1970. The study considered many options and selected the spreader bar configuration, show in Figure 1.2. The demonstration confirmed the feasibility of multilift operation for short distance. However, pilot workload was high. Therefore, in order for the system to transfer at significant velocity, automatic control is required to reduce the pilots burden. From the demonstration, the master-slave control concept was developed, where a command pilot in the master helicopter fly the formation while the slave helicopter is automatically controlled to maintain spreader bar orientation and separation. Moreover, It became apparent from the demonstration that a better understanding of twin-lift dynamics is required [3, 14, 15].



Figure 1.2. Sikorsky Twin-lift demonstration with CH-54B [3].

After the successful demonstration by Sikorsky, many research have been focus on the twin-lift system. In 1985, Cutiss and Waburton developed the equation of motion for a twin-lift system with spreader bar configuration, discovered modes of motion and nature of the control response characteristics, and suggested using feedback control to achieve a more stable system [3]. In 1986, Cicolani and Kanning provided extensive system characteristics as well as simulation results for twin-lift system [14]. In 1992 Mittal et al. investigated stability and control characteristics of two twin-lift configurations, one with spreader bar and one without. The result showed that the configuration without spreader bar is also feasible when using feedback control [5]. Mittal and Prasad also investigated three dimensional modeling and control of twin-lift system, and the research concluded that the pilot workload was very heavy during the operation so automatic control is needed to achieve a more stable operation [6]. Research into control of twin-lift systems has also been examined synthesis [16], stability augmentation using non-linear state feedback [9–11] and adaptive control [17]. In 2010, Maza et al. presented a sensor network architecture to perform cooperative mission with small multi-UAV platform, and they experimented the network with a case of three helicopter transporting a single load [7].

In 2013, Song et al. developed and implemented a multilift simulation model that is designed to be modular and generic. The model is capable of simulating various configuration of rotorcraft, spreader bars, and external loads. The model also treats each helicopter and the payload as individual object with their own equation of motion, and they only physically interact through the cable. A nonlinear control scheme was developed that uses a dynamic inversion approach to compute the desired control input such that each helicopter will closely satisfy the desired cable force requirement to control the payload. An example of four helicopters carrying a single payload was presented to demonstrate the simulation capability and the controller's performance [18].

In summary, a significant amount of research has been done in past few decades for the twin-lift system. Research for multilift beyond two rotorcraft have also been conducted in recent years [19, 20] as the sensing technology for aerial vehicle has been significantly improved in the last two decades.

### 1.3 System Overview

A control system of multilift is shown in Figure 1.3.

The "Desired Trajectory" block provides desired state of the payload, which can be automatically generated in the case of autonomous operation or from human operator in the case of manual operation. The trajectory also needs to satisfy the path constraint as well as the kinematic and dynamics constraints of the mul-



Figure 1.3. Multilift system overview.

tilfit system. OBTLC is used for the multilift system control proposed in this thesis, which contains three levels of control with trajectory following controller at top level, payload control at middle level, and rotorcraft flight controller at low level. The controller commands are send to the "Multilift Simulator" to model the dynamics of each helicopter and the payload. Note that all helicopters and the payload have their own equation of motion, and are treated as separate object instead of a single multi-body model. The "State Estimator" estimates the state of each helicopter and the payload based on sensed data from the onboard sensors.

The control layers in OBTLC work together to control the payload so that its desired path is followed and any constraints, which are necessary for operational safety and mission requirements, are satisfied. This thesis is focused on the middle level controller, "Payload Controller". It discusses the cable force computation using least-norm solution to satisfy the net force and moment and using null space solution to satisfy constraints on the system. The thesis also presents the effect of attachment point geometry (on payload) and the cable force angle constraint on the stability and controllability of the payload.

## 1.4 **Problem Description**

Upon receiving the payload's desired state, the trajectory following controller computes the desired net force and moment to "steer" the payload from its current state to the desired state. Any kind of trajectory following controller can be use for the task as long as it outputs the net force and moment on the CG of the payload (and here uses a PID controller as an example). The net force and moment and the geometry of cable attachments (on the payload) are used to compute the individual cable force for each tether, which sum up to the desired net force and moment while satisfy any constraints on the system. This step is critical as it affect controllability and stability of the payload and operation safety of the helicopter. The cable force vectors are used to compute the desired state of each cable attachment point on the helicopter. Then, the desired attachment point (on helicopter) state is sent to the helicopter flight controller to ensure that the desired cable force and direction is applied. Finally, the multilift simulator models the dynamic of each helicopter and the payload, and feeds their state back to the trajectory following controller for next time step computation. Summarizing, the multilift problem can be broken down into three basic problems: payload trajectory following, cable force computation, and helicopter control.

### 1.5 Assumptions

The following is a list of major assumptions that are considered in this thesis.

#### • Flat earth assumption

The equations of motion for both payload and helicopter are assuming that the Earth is flat and not rotating so the inertial reference frame is fixed to the Earth.

#### • Cable attachment on helicopter

For all simulations, cable is attached at the center of gravity of the helicopter to reduce the complexity of the helicopter controller since this thesis is focused on the payload control.

#### • Cable attachment point

Cable attachment points on both payload and helicopter are assumed to be frictionless ball and socket joints.

#### • Effect of cable inertia

The inertia (mass and moment of inertia) and aerodynamic effects of the cable are neglected.

## **1.6** Contributions

Primary contributions of this thesis are as follows:

#### • OBTLC design for multilift system

Provide control solution for general multilift system, which reduces the complexity by separating the control into three layers.

#### • Effect of attachment point geometry

Present analysis results on the effect of cable attachment geometry on the controllability of the system.

#### • Cable force computation

Describe method that systematically computes the cable forces to satisfy the desired net force and moment while fulfilling operational safety requirement and other constraints.

## 1.7 Reader's Guide

The remainder of this thesis is organized as follows. Chapter 2 provides an overview of the problem at hand, defining coordinates, describing the control methodology and defining dynamics of the payload. Chapter 3 discusses each of the components of the control system, focusing especially on computing the cable forces that are required for the payload to follow a desired trajectory while fulfilling constraints such as helicopter separation. Chapter 4 presents the results of analysis on cable attachment geometry and cable force constraint. Chapter 5 discusses the simulations used to show the utility of the approach, and presented their results. Chapter 6 concludes this research by summarizing the contributions of this thesis and discussing recommendations of future work.



## The Multilift Problem

The goal here is to control a load carried by N helicopters so that the payload will follow its desired trajectory (Figure 2.1). It is assumed that each helicopter has knowledge of its own state and that payload state is also known (e.g. via GPS/INS).



Figure 2.1. Schematic of coordinated slung-load transport problem.

Referring to Figure 2.1, payload position  $\mathbf{p}$  is expressed in an inertial North-East-Down frame O, and the desired trajectory is defined in this frame. Cable attachment points  $\mathbf{g}_i$  are defined in the payload-body frame, and the position  $\mathbf{r}_i$  of a helicopter is expressed in the inertial frame.

Given a desired trajectory, the required force  $\mathbf{F}_{CG}$  and moment  $\mathbf{M}_{CG}$  to ensure that the trajectory is flown can be computed from payload dynamics. The

problem is now to determine the cable forces  $\mathbf{F}_i$  so that: (1) the sum of forces and moments induced by cable forces at the payload CG is equal to the desired force and moment; (2) vehicle separation constraints are satisfied; (3) other constraints (such as controllability) are satisfied.

While trajectory generation is not the subject of this thesis, It is assumed that the desired trajectory is dynamically and kinematically feasible: i.e. each helicopter is able to generate the required thrust to maintain desired cable force, and each helicopter is equipped with a flight controller so that desired helicopter state is maintained. Issues related to helicopter control are addressed in other work [18], while this thesis is focused on the payload control especially the cable force computation of the multilift system.

## 2.1 System Description and Control Architecture

The block diagram in Figure 2.2 shows a hierarchical approach to coordinated transport.



Figure 2.2. Schematic of Object Based Task Level Control for Multilift.

The desired payload state  $\mathbf{x}^{des}$  is obtained from some payload trajectory (or human operator). A trajectory following controller computes the desired net force  $\mathbf{F}_{CG}$  and moment  $\mathbf{M}_{CG}$  acting on the payload (equivalently, the net acceleration and net angular acceleration acting about the center of gravity of the payload). Cable forces  $\mathbf{F}_i$  that result in this net force and moment are computed based on the geometry of the cable attachments and constraints such as vehicle separation. The flight controller aboard each helicopter ensures that the required cable tension and cable direction (with respect to the payload) are flown. Payload state  $\mathbf{x}$  is provided either by a sensor (e.g. GPS/INS) on the payload or it is estimated by the team of helicopters.

### 2.2 Payload Dynamics

Payload position is expressed in the North-East-Down frame as n, e, d. Payload rotations are expressed as Euler angles  $\phi, \theta, \psi$  relative to the inertial North-East-Down frame. Payload velocities u, v, w are expressed in the payload-body frame. Using the standard definition of Euler angles, payload kinematics are

$$\dot{n} = u\cos\theta\cos\psi + v(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + w(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)$$
(2.1)

$$\dot{e} =$$

 $u\cos\theta\sin\psi + v(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) + w(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)$ 

(2.2)

$$d = -u\sin\theta + v\sin\phi\cos\theta + w\cos\phi\cos\theta \tag{2.3}$$

 $\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \tag{2.4}$ 

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{2.5}$$

$$\dot{\psi} = q \frac{\sin \phi}{\cos \theta} + r \frac{\cos \phi}{\cos \theta} \tag{2.6}$$

Assuming that the payload's mass moment of inertia matrix is diagonal,

$$\dot{u} = rv - qw - g\sin\theta + \frac{F_x}{m} \tag{2.7}$$

$$\dot{v} = pw - ru + g\cos\theta\sin\phi + \frac{F_y}{m} \tag{2.8}$$

$$\dot{w} = qu - pv + g\cos\theta\cos\phi + \frac{F_z}{m}$$
(2.9)

$$\dot{p} = \frac{1}{J_x} [M_x + (J_y - J_z)qr]$$
(2.10)

$$\dot{q} = \frac{1}{J_y} [(M_y + (J_z - J_x)rp]$$
(2.11)

$$\dot{r} = \frac{1}{J_z} [M_z + (J_x - J_y)pq]$$
(2.12)

where  $\mathbf{F} = [F_x \ F_y \ F_z]^T$  (the net force acting on the payload CG, expressed in the payload-body frame) and  $\mathbf{M} = [M_x \ M_y \ M_z]^T$  (the net moment acting about the payload CG, expressed in the payload-body frame).

## 2.3 Cable Force

A cable attached to the payload at a point  $\mathbf{g}_i$  (see Figure 2.1) induces a force and moment on the CG:

$$\begin{bmatrix} \mathbf{F}_{cable,CG,i} \\ \mathbf{M}_{cable,CG,i} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{cable,i} \\ \mathbf{g}_i \times \mathbf{F}_{cable,i} \end{bmatrix}$$
(2.13)

Written as a matrix multiplication,

$$\begin{bmatrix} \mathbf{F}_{cable,CG,i} \\ \mathbf{M}_{cable,CG,i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -g_{i,z} & g_{i,y} \\ g_{i,z} & 0 & -g_{i,x} \\ -g_{1,y} & g_{1,x} & 0 \end{bmatrix} \mathbf{F}_{cable,i} = \mathbf{G}_i \mathbf{F}_{cable,i}$$
(2.14)

where  $\mathbf{g}_i = [g_{i,x} \ g_{i,y} \ g_{i,z}]^T$  defines the vector from the payload CG to the *i*<sup>th</sup> cable attachment point,  $\mathbf{G}_i$  is a geometry matrix for the *i*<sup>th</sup> cable attachment that defines the effect of the *i*<sup>th</sup> cable force on the CG, and  $\mathbf{F}_{cable,i} = [F_{cable,i,x} \ F_{cable,i,y} \ F_{cable,i,z}]^T$ are the components of the cable tension. Note that the choice of frame in which  $\mathbf{g}_i$  and  $\mathbf{F}_i$  are resolved is arbitrary (although both must be resolved in the same frame), it is in practice most convenient to resolve these in the payload-body frame.

The total force and moment acting at the payload CG is the sum of contribu-

tions from all the cables:

$$\begin{bmatrix} \mathbf{F}_{cable,CG} \\ \mathbf{M}_{cable,CG} \end{bmatrix} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{G}_2 & \cdots & \mathbf{G}_N \end{bmatrix} \begin{bmatrix} \mathbf{F}_{cable,1} \\ \mathbf{F}_{cable,2} \\ \vdots \\ \mathbf{F}_{cable,N} \end{bmatrix}$$
(2.15)
$$= \mathbf{GF}_{cable}$$
(2.16)

A six degree of freedom payload will require at least three cables to provide control over all degrees of freedom. While two cables will give six components of cable force (and thus a square **G** matrix), a solution for  $\mathbf{F}_{cable}$  will not exist because **G** will be singular. Physically, this condition will leave the payload free to rotate about the vector connecting the two attachment points. With three or more cables Equation 2.16 is underdetermined, giving an infinite number of solutions. This property will be used to compute cable forces that satisfy the desired net force and moment while simultaneously satisfying other constraints. A method for solving Equation 2.16 is discussed in a later section.

### 2.4 Helicopter Desired State and Acceleration

The components of cable force computed from the solution to Equation 2.16 define cable angles with respect to the payload-body frame. Combined with cable length, these angles define the desired position of a helicopter with respect to the payload (Assuming cable is attached at CG of the helicopter); helicopter velocity and acceleration with respect to the payload are defined by desired position and the payload trajectory. The helicopter's net thrust vector is determined by the desired cable force and by desired helicopter acceleration. This is shown schematically in Figure 2.3.

A cable is modeled as a damped spring, so the magnitude of tension in the  $i^{\rm th}$  cable is

$$F_{cable,i} = k_c \Delta l_i + c_c \dot{l}_i \tag{2.17}$$

where  $\Delta l_i$  is the difference between stretched length and unstretched length of the cable,  $k_c$  is the spring constant of the cable,  $\dot{l}_i$  is the rate of change of cable



Figure 2.3. Schematic of helicopter position with respect to CG of payload.

length, and  $c_c$  is the damping constant of the cable. Clearly cable tension must be non-negative (you can't push on a rope).

Given net cable length, the position of the  $i^{\text{th}}$  helicopter relative to the attachment point is

$$\mathbf{r}_{heli/att,i} = (l_{i,0} + \Delta l_i) \frac{\mathbf{F}_{cable,i}}{F_{cable,i}}$$
(2.18)

where  $l_{i,0}$  is the unstretched cable length. The position of a helicopter with respect to the payload CG is

$$\mathbf{r}_i^{CG} = \mathbf{g}_i + \mathbf{r}_i^{att} \tag{2.19}$$

Then the desired position, velocity, and acceleration of the  $i^{\rm th}$  helicopter relative to the inertial frame are

$$\mathbf{r}_i = \mathbf{p} + \mathbf{r}_i^{CG} \tag{2.20}$$

$$\mathbf{v}_{i} = \mathbf{v} + \dot{l}_{i} \frac{\mathbf{F}_{cable,i}}{|F_{cable,i}|} + \boldsymbol{\omega} \times \mathbf{r}_{i}^{CG}$$
(2.21)

$$\mathbf{a}_i = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r}_i^{CG} + \boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{r}_i^{CG}$$
(2.22)

Chapter

## **Coordinated Transport Control**

To illustrate the control architecture proposed in this thesis, a payload trajectory controller consisting of feedforward accelerations and state feedback is used to compute desired payload forces and moments. A general method for computing cable forces is derived: this method first computes a least norm solution for the required cable forces, and then uses the null space of the cable geometry matrix to ensure that constraints are satisfied. This guarantees that cable forces satisfy the desired net payload force and moment.

## 3.1 Payload Trajectory Control

In the transport strategy proposed here the first step is controlling payload state. This may involve maintaining position over a target or the more general case following a desired trajectory, and the specific choice of trajectory following controller is arbitrary. Recall that the output of the trajectory following controller is a desired net force and moment on the payload center of gravity (or equivalently, desired payload acceleration). For demonstration purposes a PID controller that follows a trajectory in an inertial reference frame is used here.

The payload is assumed to be near level (i.e. pitch and roll angles are small) and angular rates are small. Payload kinematics are therefore

$$\dot{n} = v_n \tag{3.1}$$

$$\dot{e} = v_e \tag{3.2}$$

$$\dot{d} = v_d \tag{3.3}$$

$$\dot{\phi} = p \tag{3.4}$$

$$\dot{\theta} = q \tag{3.5}$$

$$\dot{\psi} = r$$
 (3.6)

$$\dot{v}_n = \frac{F_n}{m} \tag{3.7}$$

$$\dot{v}_e = \frac{F_e}{m} \tag{3.8}$$

$$\dot{v}_d = g + \frac{F_d}{m} \tag{3.9}$$

$$\dot{p} = \frac{M_x}{J_x} \tag{3.10}$$

$$\dot{q} = \frac{M_y}{J_y} \tag{3.11}$$

$$\dot{r} = \frac{M_z}{J_z} \tag{3.12}$$

where  $v_{(\cdot)}$  denotes components of velocity in the north, east, or down direction and  $F_{(\cdot)}$  denotes components of the net force in the north, east or down directions. Written compactly in discrete form,

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u} \tag{3.13}$$

where

$$\mathbf{x}_{k} = [n_{k} \ e_{k} \ d_{k} \ \phi_{k} \ \theta_{k} \ \psi_{k} \ v_{n,k} \ v_{e,k} \ v_{d,k} \ p_{k} \ q_{k} \ r_{k}]^{T}$$
(3.14)

$$\mathbf{u}_{k} = \begin{bmatrix} F_{n,k} & F_{e,k} & F_{d,k} & M_{x,k} & M_{y,k} & M_{z,k} \end{bmatrix}^{T}$$
(3.15)

For trajectory following, a PID controller of the following form is used.

$$\mathbf{u}_{k} = \mathbf{K}_{p} \left( \mathbf{x}_{k,s,des} - \mathbf{x}_{k,s} \right) + \mathbf{K}_{i} \frac{1}{z-1} \left( \mathbf{x}_{k,s,des} - \mathbf{x}_{k,s} \right) + \mathbf{K}_{d} \left( \mathbf{x}_{k,d,des} - \mathbf{x}_{k,d} \right) + \mathbf{u}_{k,traj}$$
(3.16)

$$\mathbf{x}_{k,s} = \left[n_k \ e_k \ d_k \ \phi_k \ \theta_k \ \psi_k\right]^T \tag{3.17}$$

$$\mathbf{x}_{k,d} = \begin{bmatrix} v_{n,k} & v_{e,k} & v_{d,k} & p_k & q_k & r_k \end{bmatrix}^T$$
(3.18)

Where  $\mathbf{x}_{k,s,des}$  is the desired payload position and Euler angle at time step k;  $\mathbf{x}_{k,d,des}$  is the desired payload velocity and angular rate at time step k;  $\mathbf{u}_{k,traj}$  is a feed forward term computed from the desired acceleration  $\mathbf{a}_{traj}$  of payload and the payload inertia matrix **M**. A schematic of this controller is shown in Figure 3.1.



Figure 3.1. Block diagram of feedforward and state feedback controller.

The desired payload forces computed above are expressed in the inertial frame (moments are in the payload-body frame). In the body frame, payload forces and moments are

$$\begin{bmatrix} \mathbf{F}_{CG} \\ \mathbf{M}_{CG} \end{bmatrix} = \begin{bmatrix} \mathbf{T} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix} \mathbf{u}$$
(3.19)

where **T** is the direction cosine matrix that transforms a vector from the inertial frame to the payload-body frame and  $\mathbf{0}_{3\times3}$  and  $\mathbf{I}_{3\times3}$  are a 3 × 3 matrix of zeros and the identity matrix, respectively. The desired payload forces and moments can now be used to compute the desired cable forces.

## 3.2 A general formulation for cable force computation

As stated earlier, Equation 2.16 is underdetermined, hence there are an infinite number of solutions. One solution minimizes the total cable forces (the minimum norm solution), and this can be computed in closed form given a particular cable attachment geometry matrix G:

$$\mathbf{F}_{cable}^{LN} = \mathbf{G}^{T} \left( \mathbf{G} \mathbf{G}^{T} \right)^{-1} \begin{bmatrix} \mathbf{F}_{CG} \\ \mathbf{M}_{CG} \end{bmatrix}$$
(3.20)

In the payload body frame **G** is constant, thus  $\mathbf{G}^T (\mathbf{G}\mathbf{G}^T)^{-1}$  can be precomputed. Computing a set of cable forces that satisfy the desired net forces can thus happen at high rate in real time.

This least norm solution will ensure that the desired net force and moment acting on the payload center of gravity is satisfied. However, constraints (such as helicopter separation) or other considerations (such as controllability of the payload) may also apply. Cable forces that exist in the null space of  $\mathbf{G}$  will not affect payload net force and moment, and the null space can be used to satisfy constraints or other considerations. The net cable force vector will thus be

$$\mathbf{F}_{cable} = \mathbf{F}_{cable}^{LN} + \mathbf{F}_{cable}^{null} \tag{3.21}$$

where (by definition)

$$\mathbf{GF}_{cable}^{null} = 0 \tag{3.22}$$

A set of forces that satisfy Equation 3.22 can be computed as a linear combination of vectors that span the nullspace of **G**. Suppose  $\{\tilde{\mathbf{g}}_i, i = 1...3N - 6\}$  define an orthonormal basis for the nullspace of **G**, then

$$\mathbf{F}_{cable}^{null} = \begin{bmatrix} \tilde{\mathbf{g}}_1 & \tilde{\mathbf{g}}_2 & \cdots & \tilde{\mathbf{g}}_{3N-6} \end{bmatrix} \mathbf{f}$$
(3.23)

will automatically exist in the nullspace of  $\mathbf{G}$  and will thus have zero net effect on the payload center of gravity. The problem now is to find  $\mathbf{f}$  so that constraints are satisfied.

This can be done by solving the optimization problem

minimize 
$$C\left(\mathbf{F}_{cable}^{LN}, \mathbf{f}\right)$$
 (3.24)

subject to 
$$\mathbf{F}_{cable} = \mathbf{F}_{cable}^{LN} + \tilde{\mathbf{G}}\mathbf{f}$$
 (3.25)

$$g\left(\mathbf{F}_{cable}\right) \le 0 \tag{3.26}$$

$$h\left(\mathbf{F}_{cable}\right) = 0 \tag{3.27}$$

$$0 < F_i \le F_{max} \tag{3.28}$$

The procedure for computing the required cable force is thus to first compute the least norm solution and then compute the null space force so that constraints are satisfied. The cost function (Equation 3.24) defines the additional considerations (e.g. controllability) or tries to minimize total cable force; the inequality and equality constraints (Equations 3.26 and 3.27) can be use to satisfy the helicopter minimum separation constraint that ensures each helicopter is in a safe distance with respect to each other; the cable force magnitude constraint (Equation 3.28) ensures that positive tension is maintained on each cable and that maximum allowable tension is not exceeded; other constraints that improve stability and disturbance rejection of the system.

### **3.3** Helicopter control

A critical component is the helicopter's on-board control. It has two main functions: first, to ensure that the desired vehicle state is maintained; second, to ensure that the desired cable tension is maintained. Helicopter control is not the focus of this thesis, but a few criteria are briefly outlined.

The required accuracy of helicopter state control is dependent on cable length. Since the position of the helicopter with respect to the payload defines the direction of the cable force vector, a long cable will result in less sensitivity to errors in helicopter relative position. Longer cables will also result in a longer system time constant. Cable extensibility will also permit more error in helicopter state control, however the question of stability as a function of cable stiffness must be addressed.

Cable tension is critical. Loss of tension in a cable may result in loss of the payload (and of the vehicles transporting the payload). A means to measure cable tension as well as fast response to commanded changes in helicopter thrust will greatly improve overall performance.

## 3.4 Payload state computation

Here it is assumed that payload state is known through a GPS/INS mounted on the payload. If this system is not available then a means of estimating payload state from helicopter state will be required.



# Cable Attachment Geometry and Cable Force Constraint

Since the geometric matrix of cable attachments, **G**, is part of the equation of motion, the location of attachment points and the direction of cable forces will have effect on the stability and controllability of the payload. Previous work [21] showed that the effect of sudden constant wind changes with different helicopter separation constraint. This chapter looks into the problem further to obtain a more comprehensive understanding of the effect on the payload stability and controllability with different attachment geometry and cable force constraints. A 6.1 meters (20ft) standard shipping container is used for the analysis presented in this chapter. However, the methodology is same for any rectangular payload with different dimension (Ex:  $2.57m \times 1.88m \times 1.88m$  CONEX container).

## 4.1 Payload Controllability and Cable Attachment Geometry

Condition number is used to determine how the error in cable force affects the net force and net moment on the CG of the payload. When condition number is close to 1, the system is well conditioned, which means the error in cable force will have relatively the same magnitude of effect over all degree of freedoms of the payload. Condition number can be computed with singular value decomposition (SVD), where SVD for a m by n matrix  $\mathbf{M}$  is defined in Eq. (4.1) [22–24].

$$\mathbf{M} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T \tag{4.1}$$

Where  $\Sigma$  is a m by n diagonal matrix that contains square roots of the non-zero eigenvalues appeared in both  $\mathbf{M}^T \mathbf{M}$  and  $\mathbf{M} \mathbf{M}^T$ ,  $\mathbf{U}$  is the left-singular vector of  $\mathbf{M}$  (eigenvectors of  $\mathbf{M} \mathbf{M}^T$ ), and  $\mathbf{V}$  is the right-singular vector of  $\mathbf{M}$  (eigenvectors of  $\mathbf{M}^T \mathbf{M}$ ).

For linear system  $\mathbf{Mx} = \mathbf{b}$ , the condition number measures the maximum increase in the error of x with error in b. If m < n and let  $s_1, s_2, ..., s_m$  be the non-zero diagonal elements of  $\Sigma$ , then condition number of M is defined as:

$$cond(\mathbf{M}) = \frac{max(s_i)}{min(s_i)} \ge 1$$

$$(4.2)$$

For multilift system, the input force and moment on the CG of the payload is define by Eq. (2.15), and  $\mathbf{F}_{cable}$  is equal to the sum of  $\mathbf{F}_{cable}^{LN}$  and  $\mathbf{F}_{cable}^{null}$ . However,  $\mathbf{GF}_{cable}^{null} = 0$  so:

$$\begin{bmatrix} \mathbf{F}_{cable,CG} \\ \mathbf{M}_{cable,CG} \end{bmatrix} = \mathbf{G} \mathbf{F}_{cable}^{LN}$$
(4.3)

Therefore any error in least-norm part of the cable forces will have effect on the net force and moment. Using least-norm equation to revert Eq. (4.3) into:

$$\mathbf{G}^{T} \begin{bmatrix} \mathbf{G}\mathbf{G}^{T} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{F}_{cable,CG} \\ \mathbf{M}_{cable,CG} \end{bmatrix} = \mathbf{F}_{cable}^{LN}$$
(4.4)

The condition number of  $\mathbf{G}^T \left[\mathbf{G}\mathbf{G}^T\right]^{-1}$  shows how much the error in the cable force will affects the net force and net moment. The goal here is to place the attachment points at the location such that the condition number is close to 1.

By varying  $g_x$ ,  $g_y$ , and  $g_z$  of attachment point and assuming that the attachment points are symmetric (for a case of four helicopters), condition number with respect to different  $g_x$ ,  $g_y$ , and  $g_z$  is obtained. Figure 4.1 shows the minimum condition number for each  $g_z$  value over  $g_x$  and  $g_y$  values. It shows that the condition number is smaller when  $g_z$  is closer to the payload CG. Therefore, for a 6.1 meters standard shipping container that has height of about 2.6 meters, the desired z-component



Figure 4.1. Minimum condition number over  $g_x$  and  $g_y$  vs.  $g_z$ .

of attachment location is about 1.3 meters above the CG of the payload (assuming CG is at center of the container), which is the on the upper surface of the container.

Figure 4.2 is a pseudocolor plot of the condition number for  $g_z = -1.3m$  with different  $g_x$  and  $g_y$  value. The black dot on the plot shows where the minimum condition number occurred. The condition number is relatively smaller when  $g_x$  is equal to  $g_y$ , and this is where the attachment points will be placed.



Figure 4.2. Condition number over  $g_x$  and  $g_y$  for  $g_z = -1.3$  meters.

### 4.2 Cable Force Constraint

The change in the effect of sudden constant wind on the payload with respect to different helicopter separation constraint (presented in previous work [21]) is actually related to the constraint of the cable force or the null space part of the cable force. To determine how the null space of the cable force affects the stability and controllability of the payload, two analysis were performed. First analysis investigated the effect due to tension error, and second analysis investigated the effect due to cable direction error. Separating the analysis into two parts simplifies the analysis process and reveals more insight of the effect. The analysis presented in this section is for a case of four-helicopters carry a single payload, but the methodology is application to a N-helicopter scenario. To look at the effect of error in tension, cable force is rewritten into:

$$\mathbf{F}_{cable} = \begin{bmatrix} \hat{\mathbf{u}}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{u}}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{u}}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{u}}_4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \hat{\mathbf{U}}\mathbf{T}$$
(4.5)

Where  $\hat{\mathbf{u}}_1$ ,  $\hat{\mathbf{u}}_2$ ,  $\hat{\mathbf{u}}_3$ , and  $\hat{\mathbf{u}}_4$  are  $3 \times 1$  unit vectors define the direction of cable forces,  $\mathbf{0}$  is  $3 \times 1$  zeros vector, and  $T_1$ ,  $T_2$ ,  $T_3$ , and  $T_4$  are tension of the cable forces. By definition,  $\hat{\mathbf{u}}_i = [u_{x,i} \ u_{y,i} \ u_{z,i}]^T$  and  $T_i = [F_{x,i} \ F_{y,i} \ F_{z,i}]^T$ .

Linearized the equation of motion about hovering state  $\mathbf{X}_0$  and hovering input  $\mathbf{T}_0:$ 

$$\Delta \dot{\mathbf{X}} = \mathbf{A} \Delta \mathbf{X} + \mathbf{B} \mathbf{G} \hat{\mathbf{U}} \Delta \mathbf{T}$$
(4.6)

Attachment points location and cable force direction vector are defined as:

$$\mathbf{g}_{i} = \begin{bmatrix} g_{i,x} & g_{i,y} & g_{i,z} \end{bmatrix}^{T}$$

$$= \begin{bmatrix} -g_{i} \sin \alpha_{g,i} \cos \beta_{g,i} & -g_{i} \sin \alpha_{g,i} \sin \beta_{g,i} & -g_{i} \cos \alpha_{g,i} \end{bmatrix}^{T}$$

$$\mathbf{\hat{u}}_{i} = \begin{bmatrix} \hat{u}_{i,x} & \hat{u}_{i,y} & \hat{u}_{i,z} \end{bmatrix}^{T}$$

$$= \begin{bmatrix} -\sin \alpha_{i} \cos \beta_{i} & -\sin \alpha_{i} \sin \beta_{i} & -\cos \alpha_{i} \end{bmatrix}^{T}$$

$$(4.7)$$

Where  $g_i$  is the magnitude of  $\mathbf{g}_i$ ;  $\alpha_{g,i}$  is the angle between -z axis of payloadbody frame and  $\mathbf{g}_i$ ;  $\beta_{g,i}$  is the angle between  $g_{i,xy}$  and x axis of payload-body frame;  $\alpha_i$  is the angle between  $i^{\text{th}}$  cable force and -z axis of payload-body frame;  $\beta_i$ is angle between  $i^{\text{th}} F_{cable,xy}$  and x axis of payload-body frame. These definitions are illustrated in Figure 4.3



Figure 4.3. Definition of cable force angles and attachment vector angles

For diagonal inertia matrix and linearized equation of motion about hovering or cruising state ( $\phi = \theta = \psi = 0, p = q = r = 0$ ), **B** becomes:

$$\mathbf{B} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \frac{1}{m}\mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{J}^{-1} \end{bmatrix}$$
(4.9)

and  $\mathbf{B}\mathbf{G}\hat{\mathbf{U}}$  is equal to:

$$\mathbf{BG}\hat{\mathbf{U}} = \begin{bmatrix} \mathbf{0}_{6\times4} \\ \frac{1}{m}\hat{\mathbf{u}}_1 & \frac{1}{m}\hat{\mathbf{u}}_2 & \frac{1}{m}\hat{\mathbf{u}}_3 & \frac{1}{m}\hat{\mathbf{u}}_4 \\ \frac{1}{J_x}v_{1,zy} & \frac{1}{J_x}v_{2,yz} & \frac{1}{J_x}v_{3,yz} & \frac{1}{J_x}v_{4,zy} \\ \frac{1}{J_y}v_{1,xz} & \frac{1}{J_y}v_{2,xz} & \frac{1}{J_y}v_{3,zx} & \frac{1}{J_y}v_{4,zx} \\ \frac{1}{J_z}v_{1,xy} & \frac{1}{J_z}v_{2,yx} & \frac{1}{J_z}v_{3,xy} & \frac{1}{J_z}v_{4,yx} \end{bmatrix}$$
(4.10)

Where

$$\begin{aligned}
v_{i,xy} &= (g_{i,x}\hat{u}_{i,y} - g_{i,y}\hat{u}_{i,x}) \quad v_{i,yx} = (g_{i,y}\hat{u}_{i,x} - g_{i,x}\hat{u}_{i,y}) \\
v_{i,xz} &= (g_{i,x}\hat{u}_{i,z} - g_{i,z}\hat{u}_{i,x}) \quad v_{i,zx} = (g_{i,z}\hat{u}_{i,x} - g_{i,x}\hat{u}_{i,z}) \\
v_{i,yz} &= (g_{i,y}\hat{u}_{i,z} - g_{i,z}\hat{u}_{i,y}) \quad v_{i,zy} = (g_{i,z}\hat{u}_{i,y} - g_{i,y}\hat{u}_{i,z})
\end{aligned}$$
(4.11)

are scalar elements.
Equation 4.10 shows that any change in cable tension will have no effect on angular acceleration of the payload during hovering and cruising when  $\hat{\mathbf{u}}_i = \frac{\mathbf{g}_i}{g_i}$ . In this situation, state vector in Equation 3.14 becomes

$$\mathbf{x}_{k} = [n_{k} \ e_{k} \ d_{k} \ \phi_{k} \ \theta_{k} \ \psi_{k} \ v_{n,k} \ v_{e,k} \ v_{d,k} \ 0 \ 0 \ 0]^{T}.$$

Note that the attachment position  $\mathbf{g}_i$  and cable force direction  $\hat{\mathbf{u}}_i$  is arbitrary. Therefore any error in tension will not affect the attitude dynamic of the payload during hovering or cruising when cable force is in the same direction as the vector from payload CG to cable's corresponding attachment point.

Since there are four input variables, so a maximum of four degrees of freedom are controllable. Figure 4.4 shows the determinant of the outer product of controllability matrix for the controllable part of the system. Note that the cable forces are assumed to be symmetric for the result presented here. The position of attachment points used for this analysis are presented in Table 4.1. The values of the determinants have been normalized with payload mass as mass normalizer and payload length as length normalizer. Therefore the determinant presented in Figure 4.4 is unitless.

Attachment 1	$g_{1,x} = -1.5 \text{ m}$	$g_{1,y} = -1 \text{ m}$	$g_{1,z} = -1.3 \text{ m}$
Attachment 2	$g_{2,x} = -1.5 \text{ m}$	$g_{2,y} = 1 \text{ m}$	$g_{2,z} = -1.3 \text{ m}$
Attachment 3	$g_{3,x} = 1.5 \text{ m}$	$g_{3,y} = 1 \text{ m}$	$g_{3,z} = -1.3 \text{ m}$
Attachment 4	$g_{4,x} = 1.5 \text{ m}$	$g_{4,y} = -1 \text{ m}$	$g_{4,z} = -1.3 \text{ m}$

Table 4.1. Attachment position for cable force constraint analysis.



Figure 4.4. Determinant of outer product of controllability matrix for tension input.

From Figure 4.4, it is very clear that the determinant of the controllability matrix decreases as the cable force direction move closer to the direction of attachment point position (black dot), where any error in cable tension will have no effect on the attitude of the payload.

The second part is to investigate how the direction of cable force affects the controllability of the payload.

The cable force can be represented as:

$$\mathbf{F}_{cable,i} = \begin{bmatrix} T_i \sin \alpha_i \cos \beta_i & T_i \sin \alpha_i \cos \beta_i & T_i \cos \alpha_i \end{bmatrix}^T$$
(4.12)

Assuming constant tension for hovering and cruising, the input variable is defined as:

$$\boldsymbol{\Theta} = \begin{bmatrix} \alpha_1 & \beta_1 & \alpha_2 & \beta_2 & \alpha_3 & \beta_3 & \alpha_4 & \beta_4 \end{bmatrix}^T$$
(4.13)

Again, the angle  $\alpha_i$  and  $\beta_i$  is defined in Figure 4.3.

Linearizing the equation of motion about hovering state  $\mathbf{X}_0$  and reference input  $\mathbf{\Theta}_0 = [\alpha_{1,0}, \beta_{1,0}, \dots \alpha_{4,0}, \beta_{4,0}]$  with constant  $T_i$ , the system equation becomes:

$$\Delta \dot{\mathbf{X}} = \bar{\mathbf{A}} \Delta \mathbf{X} + \bar{\mathbf{B}} \Delta \boldsymbol{\Theta} \tag{4.14}$$

Same attachment point position presented in Table 4.1 is used for this analysis. Figure 4.5 shows the log scale of determinant of the outer product of controllability matrix.



Figure 4.5. log scale of determinant of the outer product of controllability matrix for direction input.

From Figure 4.5, the payload become more controllable as the cable force spreading outward. This result matches the intuition. However the cost for a more stable payload is higher cable tension, which will result in higher fuel consumption for the helicopter. Therefore, helicopter payload capacity and fuel consumption will have to be considered when constraining the cable force vector.



## **Modeling and Simulation**

This chapter describes the multilift simulation modeling and presents simulation results to show the utility of proposed approach.

### 5.1 Multilift Simulation Modeling

While the control methodology described in Chapter 3 is scalable to (in principle) any size of flock, here a group of four helicopters cooperatively transporting a payload is discussed as the motivating example (Section 5.2.4 describes a case of six-helicopter payload transport.).

Figure 5.1 shows a block diagram of the simulation. Each helicopter is connected to the payload by a cable (modeled as a damped spring).

### 5.1.1 Helicopter Simulation Model

The focus of this thesis is on payload control, thus a point mass model of the aircraft translational kinematics is coupled with a linear representation of the attitude and thrust dynamics. It is assumed that a dynamic inversion control law regulates the attitude and thrust dynamics to follow a linear command filter. An outer loop dynamic inversion control law was designed to track the desired position, velocity, and acceleration commands of each helicopter. Figure 5.2 shows the free body diagram of the helicopter. Where  $F_c$  is the cable force;  $F_T$  is the thrust;  $m_h$  is the mass of the helicopter.



Figure 5.1. Multilift simulation overview.



Figure 5.2. Free body diagram for helicopter.

Helicopter dynamics are second order in the closed loop:

$$\begin{bmatrix} \ddot{n}_i \\ \ddot{e}_i \\ \ddot{d}_i \end{bmatrix} = \frac{1}{m} \left( f(\phi_c, \theta_c, \psi_c, T_c) - \frac{1}{2}\rho f_e v_a \begin{bmatrix} \dot{n}_i \\ \dot{e}_i \\ \dot{d}_i \end{bmatrix} - \mathbf{F}_{cable} \right)$$
(5.1)

Inputs are Euler angles and total thrust, with Euler angles are filtered through a second order command filter and thrust filtered through a first order command filter.

$$\ddot{\phi}_c + 2\zeta_\phi \omega_\phi \dot{\phi}_c + \omega_\phi^2 (\phi_c - \phi_{cmd}) = 0$$
(5.2)

$$\ddot{\theta}_c + 2\zeta_\theta \omega_\theta \dot{\theta}_c + \omega_\theta^2 (\theta_c - \theta_{cmd}) = 0$$
(5.3)

$$\ddot{\psi}_c + 2\zeta_\psi \omega_\psi \dot{\psi}_c + \omega_\psi^2 (\psi_c - \psi_{cmd}) = 0$$
(5.4)

$$\tau_T \dot{T}_c + (T_c - T_{cmd}) = 0 \tag{5.5}$$

Filter parameters are dependent on the inner loop control law bandwidth, which in turn is limited by the dynamics of the specific vehicle. In the following simulations, bandwidth values were selected to match typical values for full scale rotorcraft (here Kaman K-MAX is used).

 $\phi_{cmd}$ ,  $\theta_{cmd}$ , and  $T_{cmd}$  are computed from desired position, velocity, and acceleration using an outer loop inversion controller.

$$\begin{bmatrix} \phi_{cmd} \\ \theta_{cmd} \\ T_{cmd} \end{bmatrix} = \mathbf{L}^{-1} \mathbf{C}(s) + \begin{bmatrix} \phi_c \\ \theta_c \\ 0 \end{bmatrix}$$
(5.6)

Where

$$\mathbf{L} = \begin{bmatrix} -T_c(-S\phi_c S\theta_c C\psi_c + C\phi_c S\psi_c) & -T_c C\phi_c C\theta_c C\psi_c & -C\phi_c S\theta_c C\psi_c - S\phi_c S\psi_c \\ -T_c(-S\phi_c S\theta_c S\psi_c - C\theta_c C\psi_c & -T_c C\phi_c C\theta_c S\psi_c & -C\phi_c S\theta_c S\psi_c + S\phi_c C\psi_c \\ T_c S\phi_c C\theta_c & T_c C\phi_c S\theta_c & -C\phi_c C\theta_c \end{bmatrix}$$
(5.7)

$$S\phi_c = \sin\phi_c \quad S\theta_c = \sin\theta_c \quad S\psi_c = \sin\psi_c$$
$$C\phi_c = \cos\phi_c \quad C\theta_c = \cos\theta_c \quad C\psi_c = \cos\psi_c$$

$$\mathbf{C}(s) = \begin{bmatrix} K_{P,x} \dot{e}_x + K_{I,x} e_x + K_{D,x} \ddot{e}_x \\ K_{P,y} \dot{e}_y + K_{I,y} e_y + K_{D,y} \ddot{e}_y \\ K_{P,z} \dot{e}_z + K_{I,z} e_z + K_{D,z} \ddot{e}_z \end{bmatrix}$$
(5.8)

$$\ddot{e} = a_{cmd} - a$$
$$\dot{e} = v_{cmd} - v$$
$$e = p_{cmd} - p$$

 $\psi_{cmd}$ , is computed from payload desired yaw angle and velocity direction.

$$\psi_{cmd} = \psi_{des,load} + tan^{-1} \left(\frac{v_{des,load}}{u_{des,load}}\right)$$
(5.9)

### 5.1.2 Cable Simulation Model

The cable is modeled as spring damper system in the multilift simulation. However, unlike a spring damper system, the cable is modeled such that it has no resistance to compression. Note that there may be damping due to energy dissipation when the cable is retracting while still in tension, which adds more damping to the system. This additional damping is neglected in this model for a more conservative assumption. The magnitude of the cable force is

$$F_{cable} = F_{spring} + F_{damp} \tag{5.10}$$

where

$$F_{spring} = \begin{cases} K_c \Delta l, & \text{if } \Delta l > 0\\ 0, & \text{if } \Delta l \le 0 \end{cases}; \quad F_{damp} = \begin{cases} C_c \dot{l}, & \text{if } \dot{l} > 0\\ 0, & \text{if } \dot{l} \le 0 \text{ or } \Delta l \le 0 \end{cases}$$
(5.11)

The direction of the cable force is computed from the position of the helicopter

relative to the cable attachment point on the payload.

#### 5.1.3 Payload Simulation Model

A CONEX container is used as the payload for the simulation. The dynamics of payload is modeled as a rigid body with six degree of freedom using flat Earth assumption. The model uses the non-linear equations of motion that presented in Equation 2.1 to Equation 2.12, and these equations can be represented in compactly as:

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{F}_{net,CG}, \mathbf{M}_{net,CG}) \tag{5.12}$$

where

$$\mathbf{F}_{net,CG} = \mathbf{F}_{cable,CG} + \mathbf{F}_{aero,CG} + \mathbf{F}_{gravity,CG}$$
(5.13)

$$\mathbf{M}_{net,CG} = \mathbf{M}_{cable,CG} + \mathbf{M}_{aero,CG}$$
(5.14)

are net force and moment on the payload CG.  $\mathbf{F}_{cable,CG}$  represents the resultant cable force;  $\mathbf{F}_{aero,CG}$  is the aerodynamics force on the payload as it is transporting though air;  $\mathbf{F}_{gravity,CG}$  represents force due to gravity;  $\mathbf{M}_{cable,CG}$  is the resultant moment due to cable forces;  $\mathbf{M}_{aero,CG}$  is the aerodynamics moment about the CG of payload.

The aerodynamics characteristics of a CONEX container with main dimension of 2.57m x 1.88m x 1.88m were studied in wind tunnel tests. Experimental data of the force and moment coefficients with respect to side slipping angle and angle of attach were presented in the paper [25]. This high fidelity aerodynamics model is used in the multilift simulation to obtain realistic behavior of the payload.

### 5.1.4 Dryden Wind Turbulence Model [1]

A simplified Dryden Wind Turbulence Model that uses Military Specification MIL-F-8785C is used to model the turbulence wind on the payload. The simplified model only simulates the velocity of the turbulent wind at altitude below 305 meters (angular velocity is not considered here). This model generates turbulence signal by passing band-limited white noise through forming filters. The transfer function of the forming filters are presented in following

$$H_u(s) = \alpha_u \sqrt{\frac{2L_u}{\pi V}} \frac{1}{1 + \frac{L_u}{V}s}$$
(5.15)

$$H_v(s) = \alpha_v \sqrt{\frac{L_v}{\pi V}} \frac{1 + \frac{\sqrt{3}L_v}{V}s}{(1 + \frac{L_v}{V}s)^2}$$
(5.16)

$$H_w(s) = \alpha_w \sqrt{\frac{L_w}{\pi V}} \frac{1 + \frac{\sqrt{3}L_w}{V}s}{(1 + \frac{L_w}{V}s)^2}$$
(5.17)

Where V is air speed of the payload (in unit of ft/s);  $L_{(\cdot)}$  and  $\alpha_{(\cdot)}$  are turbulence scale length and turbulence intensities respectively, which are altitude dependence. For altitude below 305 meters (1000ft), they are defined as:

$$L_u = L_v = \frac{h}{(0.177 + 0.000823h)^{1.2}}$$
(5.18)

$$L_w = h \tag{5.19}$$

$$\alpha_w = 0.1 W_{20} \tag{5.20}$$

$$\alpha_u = \alpha_v = \frac{1}{(0.177 + 0.000823h)^{0.4}} \tag{5.21}$$

Where h is payload altitude (in unit of ft), and  $W_{20}$  is wind speed at altitude of 6 meters (in unit of ft/s). Typically for light turbulence,  $W_{20} = 7.72m/s$ ; for moderate turbulence,  $W_{20} = 15.4m/s$ ; for strong turbulence,  $W_{20} = 23.15m/s$ . Figure 5.3 is an example of turbulence on the payload generated by the Dryden wind model with V = 20m/s, h = 20m, and  $W_{20} = 15m/s$ .



Figure 5.3. Wind turbulence generated by Dryden Wind Turbulence Model.

### 5.1.5 Constraints

The cost function for cable force computation, defined in Equation 5.22, will try to minimize the total cable force, which will minimize fuel consumption for the helicopter.

$$C(F_{cable}^{LN}, f) = \sqrt{F_{cable,1}^2 + F_{cable,2}^2 + F_{cable,3}^2 + F_{cable,4}^2}$$
(5.22)

where  $F_{cable,i}$  is defined by Equation 3.21.

Helicopter minimum separation constraint (26 meters) is applied to ensure each helicopter is in a safe distance with respect to each other, and maximum cable force constraint is applied to ensure the cable tension for each attachment point is smaller than helicopter's external load capacity.

Cable force angle constraint is also applied to maintain a desired angle between the cable force vector and -z axis of payload-body frame. This constraint is used to improve payload stability, and it is defined in Equation 5.23.

$$(-\hat{z}) \cdot \frac{\mathbf{F}_{cable,i}}{F_{cable,i}} = \cos \alpha_i$$
 (5.23)

Where  $-\hat{z}$  is the unit vector of -z axis of the payload-body frame;  $\mathbf{F}_{cable,i}$  is the  $i^{\text{th}}$  cable force vector represented in payload-body frame;  $F_{cable,i}$  is the tension of  $i^{\text{th}}$  cable force;  $\alpha_i$  is the constraint angle.

### 5.2 Simulation Results

Result of four simulations are presented in this section to demonstrate the utility of the proposed method. In all simulations, the Kaman K-MAX helicopter is used as a representative autonomous helicopter, and some of its parameters are presented in Table A.1. Attachment position for all four-helicopter simulation is shown in Table A.2, and attachment location for six-helicopter simulation is presented in Table A.6. Other simulation parameters are also presented in Appendix A

Note that "stability" defined in Section 5.2.1 and Section 5.2.2 refers to a somewhat stricter definition of loss of cable tension: if tension in one of the cables drops to zero (or below), loss of stability is assumed to occur. Note that in a physical implementation this is not necessarily true: the team of rotorcraft may be able to compensate for a loss of tension in one cable if the remaining helicopters are able to maintain controlled flight with the additional load that would result from one (or more) of the team failing to maintain tension. However, the stricter definition of "stability=positive definite cable tension" is conservative and easy to quantify.

#### 5.2.1 Monte Carlo Simulation: Cable Angle

Result of the first Monte Carlo simulation for a four-helicopter system is presented in this section. This simulation investigated the effect of turbulence wind disturbance on the payload with respect to different cable force constraint angles. A simplified Dryden wind turbulence model defined in Section 5.1.4 is used to simulate the turbulence disturbance on the payload. The payload is commanded to hold its position at [0, 0, -10m] while a constant wind is applied on the payload to simulate air speed (V) of the payload (which generates turbulence). In this simulation, the cable angle is ranged from 33 degrees to 39 degrees; wind speed (result in payload air speed, V) is ranged from 2m/s to 20m/s; Other parameters are presented in Table A.3. Samples are taken from 20 runs each with different turbulence noise seed. Payload stability result is presented in Figure 5.4.

	Payload Stability									
N ( (										
v (m/s)		33	34	35	36	37	38	39		
2	Mean	1	0	0	0	0	0	0		
2	Std	0	0	0	0	0	0	0		
Л	Mean	1	0	0	0	0	0	0		>00
4	Std	0	0	0	0	0	0	0		2 0.0
6	Mean	1	0	0	0	0	0	0		
0	Std	0	0	0	0	0	0	0		> 0 025
8	Mean	1	0.05	0	0	0	0	0		/ 0.023
0	Std	0	0.1539	0	0	0	0	0		
10	Mean	1	0.45	0	0	0	0	0		>01
10	Std	0	0.394	0	0	0	0	0		20.1
12	Mean	1	0.775	0	0.05	0	0.025	0		
12	Std	0	0.3024	0	0.2236	0	0.1118	0		>03
1/	Mean	1	0.925	0.35	0.175	0.25	0.25	0.325		20.5
14	Std	0	0.1832	0.3663	0.2936	0.3804	0.4136	0.4375		
16	Mean	1	1	0.8	0.375	0.375	0.325	0.45		> 0 F
10	Std	0	0	0.2991	0.4253	0.4552	0.4667	0.484		20.5
10	Mean	1	1	0.975	0.925	0.875	0.825	0.9		
10	Std	0	0	0.1118	0.1832	0.3193	0.3726	0.3078		
20	Mean	1	1	0.975	1	1	1	1		
20	Std	0	0	0.1118	0	0	0	0		

Figure 5.4. Stability of the payload for cable angle Monte Carlo.

In Figure 5.4, value '0' indicates the payload is stable throughout the simulation; value '0.5' indicates cable lost occurred but payload remained stable before simulation ended (could be unstable for longer simulation); value '1' indicates payload became unstable before simulation ended.

Results for average position and attitude errors are presented in Figure 5.5 and Figure 5.6; value of '1000' is assigned to average error when payload is unstable. Also,  $\alpha_{g,i}$  is equal to 35 degrees in this simulation.

			Average	e Position	Error (met	ter)		
V (m (a)				Cabl	e Angle (de	gree)		
v (m/s)		33	34	35	36	37	38	39
2	Mean	1000	0.4185	0.2308	0.2115	0.2028	0.2046	0.2019
2	Std	0	0.1078	0.0594	0.0429	0.0341	0.0306	0.0306
4	Mean	1000	0.7345	0.377	0.3247	0.2981	0.2931	0.2849
4	Std	0	0.2849	0.1442	0.1199	0.1057	0.1002	0.0951
6	Mean	1000	0.7884	0.437	0.3667	0.3323	0.3249	0.3148
0	Std	0	0.1858	0.1044	0.0815	0.0684	0.0616	0.0577
Q	Mean	1000	1.1784	0.5982	0.4769	0.4244	0.4023	0.3763
0	Std	0	0.4381	0.2094	0.1301	0.111	0.0956	0.086
10	Mean	1000	251.23	0.8597	0.6509	0.5683	0.5269	0.5056
10	Std	0	443.53	0.2914	0.1533	0.1261	0.1145	0.1213
12	Mean	1000	600.84	1.1346	50.7648	0.6855	0.6486	0.6039
12	Std	0	501.57	0.2252	223.3298	0.1001	0.1045	0.1165
1/	Mean	1000	850.3	151.2601	51.1785	151.0223	200.83	250.8538
14	Std	0	365.61	365.8047	223.3298	365.9071	409.966	443.756
16	Mean	1000	1000	650.6653	251.2144	301.0512	300.85	400.8715
10	Std	0	0	488.4304	443.5426	469.4566	469.59	501.8949
19	Mean	1000	1000	950.1003	850.3699	850.2419	800.348	900.1592
10	Std	0	0	223.1584	365.4442	365.7568	409.678	307.3036
20	Mean	1000	1000	950.1334	1000	1000	1000	1000
20	Std	0	0	223.0102	0	0	0	0

Figure 5.5. Average position error of the payload for cable angle Monte Carlo.

	Average Euler Angular Error (degree)								
V (m /a)				Cabl	e Angle (de	gree)			
		33	34	35	36	37	38	39	
2	Mean	1000	0.4057	0.3125	0.2778	0.2579	0.2539	0.2448	
2	Std	0	0.2228	0.1867	0.1688	0.1571	0.1564	0.1501	
Л	Mean	1000	0.764	0.6126	0.5485	0.5092	0.5007	0.48	
4	Std	0	0.4792	0.3927	0.3481	0.3185	0.3137	0.297	
6	Mean	1000	0.8298	0.6748	0.6122	0.5735	0.5659	0.5457	
0	Std	0	0.2821	0.245	0.2253	0.2125	0.2133	0.2055	
Q	Mean	1000	1.1524	0.9497	0.887	0.8461	0.8462	0.8214	
0	Std	0	0.4225	0.3606	0.3438	0.3284	0.3301	0.3188	
10	Mean	1000	251.37	1.4178	1.3395	1.2818	1.2876	1.2476	
10	Std	0	443.45	0.4567	0.4436	0.4266	0.4279	0.4142	
12	Mean	1000	600.93	1.6536	51.4823	1.5091	1.5216	1.488	
12	Std	0	501.47	0.3682	223.2582	0.3423	0.348	0.3416	
14	Mean	1000	850.36	152.009	53.2404	151.9119	201.798	251.803	
14	Std	0	365.48	365.4821	222.8918	365.5239	409.4694	443.1949	
16	Mean	1000	1000	651.1189	252.4463	301.874	301.8422	401.566	
10	Std	0	0	487.797	442.816	468.9038	468.9252	501.313	
10	Mean	1000	1000	950.1707	850.5633	301.874	800.5564	900.2671	
10	Std	0	0	222.8435	364.9718	365.3019	409.2496	306.9713	
20	Mean	1000	1000	950.1334	1000	1000	1000	1000	
20	Std	0	0	223.0102	0	0	0	0	

Figure 5.6. Average attitude error of the payload for cable angle Monte Carlo.

In Figure 5.4 to Figure 5.6, payload is more stable and error is smaller when angle of cable force  $(\alpha_i)$  is greater than or equal to the angle of attachment point  $(\alpha_{g,i})$ , which is equal to 35 degrees in this case. Figure 5.7 illustrates the situations when  $\alpha_i < \alpha_{g,i}$ ,  $\alpha_i = \alpha_{g,i}$ , and  $\alpha_i > \alpha_{g,i}$ .



Figure 5.7. Different cable angle constraints.

In Figure 5.7, payload CG is above the vertex point formed by the cable force vectors when  $\alpha_i < \alpha_{g,i}$ . This is similar to an inverted pendulum system, which is unstable under disturbance. Therefore, the payload is less stable and error is larger in this situation. When  $\alpha_i = \alpha_{g,i}$  (Column with red border in Figure 5.4 to Figure 5.6), cable forces do not have control on pitch and roll of the payload so any error in cable tension will not affect pitch and roll dynamics of the payload. Therefore, the payload is under disturbance, cable forces will be off the constraint to control the payload is under disturbance, cable forces will be off the payload. In the case of  $\alpha_i > \alpha_{g,i}$ , the payload CG is below the vertex point formed by cable force vectors, which is similar to a normal pendulum system. Therefore the

payload is more stable. These results match the analysis results in Chapter 4.

### 5.2.2 Monte Carlo Simulation: Cable Stiffness

Result of the second Monte Carlo simulation for a four-helicopter system is presented in this section. This simulation investigated the effect of turbulence wind disturbance on the payload with respect to different cable stiffness. The Young's modulus of the cable is ranged from 1000 to 60000 MPa while cable force angle  $(\alpha_i)$  is constrained at 37 degrees. The payload is also commanded to hold its position at [0, 0, -10m] while a constant wind is applied on the payload to simulate air speed, V. Other parameters for this simulation are presented in Table A.4. Again, samples are taken from 20 runs each with different turbulence noise seed. Payload stability result is presented in Figure 5.8.

			F	Payload St	ability					
N ( ( )			1							
v (m/s)		1000	2000	5000	2000	15000	30000	60000	1	
2	Mean	0	0	0	0	0	0	1	İ	
Z	Std	0	0	0	0	0	0	0		
4	Mean	0	0	0	0	0	0	1		>0
4	Std	0	0	0	0	0	0	0		≥0.
6	Mean	0	0	0	0	0	0	1		
0	Std	0	0	0	0	0	0	0		>00
0	Mean	0	0	0	0	0	0	1		20.0
0	Std	0	0	0	0	0	0	0		
10	Mean	0	0	0	0	0	0.05	1		>0
10	Std	0	0	0	0	0	0.2236	0		20.
12	Mean	0	0	0	0	0	0.2	1		
12	Std	0	0	0	0	0	0.4104	0		> 0
14	Mean	0.05	0	0	0.075	0.25	0.75	1	i i	20.
14	Std	0.1539	0	0	0.2447	0.3804	0.3804	0		
16	Mean	0.15	0.05	0.025	0.075	0.375	0.95	1	i i	> 0
10	Std	0.3285	0.2236	0.1118	0.2447	0.4552	0.1539	0		>0.
10	Mean	0.525	0.475	0.45	0.55	0.875	1	1		
18	Std	0.4993	0.4723	0.484	0.484	0.3193	0	0		
20	Mean	0.725	0.675	0.75	0.85	1	1	1		
20	Std	0.4435	0.4667	0.4136	0.2856	0	0	0		

Figure 5.8. Stability of the payload for cable stiffness Monte Carlo.

Again, value "0" indicates the payload is stable; value "0.5" indicates cable tension lose occurred, but payload did not became unstable; value "1" indicates the payload became unstable before the simulation ended. Figure 5.9 and Figure 5.10 shows the average error of the payload position and attitude respectively.

	Average Position Error (meter)									
V (m (a)		Cable Young's Modulus (MPa)								
v (m/s)		1000	2000	5000	2000	15000	30000	60000		
2	Mean	0.0861	0.0938	0.118	0.1622	0.2028	0.3426	1000	1	
2	Std	0.0306	0.0319	0.0343	0.0365	0.0341	0.0318	0		
4	Mean	0.1621	0.1736	0.2076	0.2605	0.2981	0.4443	1000		
4	Std	0.0726	0.0773	0.0896	0.1035	0.1057	0.1163	0		
6	Mean	0.1854	0.1982	0.2356	0.2925	0.3323	0.4915	1000		
0	Std	0.0464	0.0498	0.058	0.0664	0.0684	0.0786	0		
0	Mean	0.2641	0.2792	0.3197	0.382	0.4244	0.5762	1000		
0	Std	0.0773	0.0801	0.0873	0.1004	0.111	0.1199	0		
10	Mean	0.3402	0.3614	0.4191	0.509	0.5683	50.736	1000		
10	Std	0.0693	0.0686	0.082	0.1118	0.1261	223.4337	0		
12	Mean	0.4184	0.4482	0.5241	0.6302	0.6855	200.7698	1000		
12	Std	0.063	0.0669	0.075	0.0903	0.1001	409.9965	0		
14	Mean	0.5438	0.5675	0.678	50.8039	151.0223	650.4479	1000		
14	Std	0.1347	0.1234	0.1351	223.4176	365.9071	488.7342	0		
16	Mean	100.6073	50.6879	0.8594	51.0615	301.0512	900.1929	1000		
10	Std	307.5858	223.4449	0.1691	223.3572	469.4566	307.1997	0		
18	Mean	500.3769	400.5215	400.6789	500.6921	850.2419	1000	1000		
10	Std	512.6025	502.1879	502.056	512.2791	365.7568	0	0		
20	Mean	700.303	650.3418	700.3103	750.4119	1000	1000	1000		
20	Std	469.6874	488.8825	469.6761	443.5297	0	0	0		

Figure 5.9. Average position error of the payload for cable stiffness Monte Carlo.

	Average Euler Angular Error (degree)									
V (m (a)				Cable Yo	ung's Modu	lus (MPa)			ľ	
		1000	2000	5000	2000	15000	30000	60000		
	Mean	0.2601	0.2626	0.2651	0.267	0.2579	0.2702	1000		
2	Std	0.1642	0.164	0.1643	0.1646	0.1571	0.1655	0		
4	Mean	0.5196	0.5229	0.526	0.5283	0.5092	0.5313	1000		>0.0
4	Std	0.3322	0.334	0.3354	0.3357	0.3185	0.3342	0		≥ 0.0
6	Mean	0.5843	0.587	0.5903	0.5931	0.5735	0.5975	1000		
0	Std	0.2236	0.2235	0.2234	0.2232	0.2125	0.2238	0		> 0.75
Q	Mean	0.8749	0.8754	0.8769	0.8791	0.8461	0.8821	1000		20.75
0	Std	0.3409	0.341	0.3425	0.3441	0.3284	0.3441	0		
10	Mean	1.3288	1.3299	1.3318	1.3349	1.2818	51.2274	1000		>10
10	Std	0.4391	0.4398	0.441	0.443	0.4266	223.3182	0		/ 1.0
12	Mean	1.5689	1.566	1.5646	1.5651	1.5091	201.1995	1000		
12	Std	0.3588	0.3591	0.3577	0.3558	0.3423	409.7761	0		>20
14	Mean	2.5554	2.3567	2.3097	52.1599	151.9119	650.6005	1000		/ 2.0
14	Std	1.0636	0.6334	0.5981	223.0991	365.5239	488.5209	0		
16	Mean	102.6652	52.5775	2.713	52.5578	301.874	900.383	1000		> 10 0
10	Std	306.8828	223.0005	0.4782	223.0051	468.9038	306.6156	0		/ 10.0
10	Mean	501.6855	401.9782	402.7256	501.4413	301.874	1000	1000		_
10	Std	511.2601	500.9679	500.3517	511.5106	365.3019	0	0		
20	Mean	701.1332	651.065	700.8497	750.8782	1000	1000	1000		
20	Std	468.3873	487.8715	468.8308	442.7013	0	0	0		

Figure 5.10. Average attitude error of the payload for cable stiffness Monte Carlo.

In Figure 5.8 to Figure 5.10, payload is more stable and error is smaller when cable is less stiff (smaller Young's Modulus). This result matches the intuition discussed in Chapter 3, Section 3.3. For the same magnitude of error in helicopter states, a "soft" cable will result in less cable force error on the payload while a "stiff" cable will cause larger acceleration error on the payload that is more likely to cause cable tension lost as the payload accelerated "too fast" toward the helicopters.

### 5.2.3 Transportation: Four-Helicopter

Results for a four-helicopter system transportation are presented here to demonstrate the utility of proposed method in this thesis. Parameters used in this simulation are presented in Table A.5. Figure 5.11 shows the desired state of the payload for this simulation, and Figure 5.12 to Figure 5.14 show the results of this simulation.



Figure 5.11. Desired payload state for four-helicopter transport.



Figure 5.12. Payload state errors for four-helicopter transport.



Figure 5.13. Cable tension for four-helicopter transport.



Figure 5.14. Helicopter relative distance for four-helicopter transport.

As shown in Figure 5.11, the payload is first been lifted up to a height of 250 meters and then transported to a distance of 3225 meters toward the north. When the payload arrived at its destination, it holds its position while changing its yaw angle from 0 degrees to 45 degrees. After the yaw rotation is completed, the payload holds its final state for 60 seconds.

Figure 5.12 shows the payload state error. The error plot is noisy due the the noisy input of the helicopter and turbulence disturbance. However, the payload state error is bounded with maximum translational error less than two meters and maximum rotational error less than two degrees. Figure 5.13 shows the cable tension on each helicopter. The maximum cable tension is about 23500 N on helicopter 4, which is smaller than the load capacity of the K-MAX, 26703N (green line in the plot). Figure 5.14 shows the relative distance between each helicopter. The smallest separation distance between two helicopter is about 43.5 meters, which is much larger than the minimum required separation distance (26 meters). This result shows that proposed approach have done a very good job controlling the payload to follow its desired trajectory while satisfying the helicopter separation constraint and maximum tension constraint for safety operation. Other results for this simulation are shown in Appendix B

#### 5.2.4 Transportation: Six-helicopter

The proposed method can easily be expanded to a case of six helicopters, where two cables and two helicopters are added to the system. Table A.6 shows the position of attachment points in payload-body frame, and Table A.7 shows the simulation parameters for this case. Figure 5.15 to Figure 5.17 show the results of this simulation, and the desired trajectory of the payload is the same as the four-helicopter transportation case.

Figure 5.15 shows the payload state error. The error plot is also noisy due to the noisy helicopter input and turbulence. The payload state error is bounded with maximum translational error less than 1.5 meters and maximum rotational error less than two degrees.



Figure 5.15. Payload state errors for six-helicopter transport.



Figure 5.16. Cable tension for six-helicopter transport.



Figure 5.17. Helicopter relative distance for six-helicopter transport.

Figure 5.16 shows the cable tension on each helicopter. The maximum cable tension is about 24000 N on helicopter 4. In Figure 5.17, the smallest separation distance between two helicopters is about 31 meters, which is larger than

the minimum required separation distance. This result shows that the proposed approach is scalable to allow the team of helicopter to grow for heavier payload while maintaining its performance. Other results for this simulation are shown in Appendix B

# Chapter 6

# Conclusion

This thesis is motivated by the economical efficiency of using multilift to transport a single heavy payload. Twin-lift system has been investigated in many research over past few decades. The utility of multilift concept can be fully augment with a more general solution that is scalable beyond two helicopters. This thesis uses OBTLC to develop a general solution for multilift focusing on the payload control layer that computes the cable force to control the payload to follow its desired trajectory while satisfying other constraints for operational safety and mission requirements.

OBTLC was developed for robot manipulator system, and has been used extensively in this field. The framework divides a complex control into multiple layers of control that are theoretically independent from each other. In the application of multilift presented in this thesis, the top-level control is a trajectory following controller (PID), the mid-level is payload control that computes cable force to satisfy desired acceleration and constraints, and the low-level control is controller on board each helicopter that ensures the required cable tension and direction is satisfied. Within the OBTLC, this thesis focused on the mid-level control that uses a two steps approach to compute the cable force: first, the least norm solution ensures that the desired net force and moment are satisfied; second, null space cable force are computed so that constraints (here, vehicle separation and cable force angle constraint) are satisfied. This approach requires three or more cable attachment points on the payload to fully control the six dynamical degrees of the payload. Attachment location analysis reveals that the condition number is smaller when z-component is closer to payload CG and x-component is close to y-component. Moreover, cable force constraint analysis shows that error in cable tension will have no effect on payload's attitude dynamics when all the cable force vectors are in the same direction with their corresponding attachment point position vector. The analysis also shows that payload is more controllable when cable force vectors are more spread out.

Simulation results of payload transport using four helicopters shows the utility of the approach. payload state error remains small and disturbance rejection (response to a step gust and turbulence) is good. The six-helicopter simulation result shows that the proposed solution not only able to control the system for stable and safe operation, but also allow the number of helicopter to increase as the payload mass increases. The cable stiffness Monte Carlo simulation result matched the intuition, and cable angle Monte Carlo simulation results verified the result in cable force constraint analysis.

### 6.1 Summary of Contributions

### 6.1.1 OBTLC Design for Multilift System

An control approach using OBTLC was developed for a general multilift system with payload trajectory following controller at top-level, cable force computation at mid-level, and rotorcraft control at low-level. These layers of control work together to ensure the payload follow its desired trajectory while satisfy other constraints. A big advantage of this approach is that the number of rotorcraft is expandable for heavier payload.

### 6.1.2 Attachment Point Geometry and Cable Force Constraint

The method used for the cable attachment geometry and cable force constraint analysis can also apply to other rectangular payload to determine the optimal attachment point location for that payload and desire cable force angle constraint for the cable.

### 6.1.3 Cable Force Computation

A two-step cable force computation method was developed that first computes the least-norm solution to satisfy the net force and moment from the trajectory following controller and then uses null space of cable force to satisfy other constraints on the system.

### 6.2 Recommendations for Future Work

### 6.2.1 Helicopter Controller

More research can be done to optimize the helicopter controller so that the desired cable force can be more accurately followed. This will provide a better control on the payload and further prevent cable tension lost.

### 6.2.2 Hardware Implementation

Even though the approach proposed in this thesis was validated in the simulation, actual hardware test will provide more insight into their validity. Hardware test will expose the control to actual multilift system and define its true performance. A small scale test can be perform indoor using small helicopter. Motion capture technology can be use for position and attitude determination; IMU can be use to determine acceleration of the payload; flex sensor can be use to determine the cable tension on each attachment point; relative position between helicopters and the payload can be use to determine the cable direction.



# **Simulation Parameter**

K-MAX								
Helicopter Mass (kg)	2720.6							
Payload capacity (kg)	2722	Rotor diameter (m)	8.4					
Length $(m)$	15.8	Height (m)	4.14					
$\omega_{\phi},  \omega_{ heta}$	4  rad/s	$\omega_\psi$	1  rad/s					
$\zeta_{\phi},\ \zeta_{ heta},\ \zeta_{\psi}$	0.9	$ au_T$	0.25					
angular noise variance $(deg^2)$	$5^{2}$	angular noise mean $(deg)$	0					
thrust noise variance $(N^2)$	$2668.9^2$	thrust noise mean $(N)$	0					

 Table A.1. K-MAX helicopter parameters used in all simulations [4].

 Table A.2. Attachment position for four-helicopter simulations.

Attachment 1	$g_{1,x} = -0.4654 \text{ m}$	$g_{1,y} = -0.4654 \text{ m}$	$g_{1,z} = -0.94 \text{ m}$
Attachment 2	$g_{2,x} = -0.4654 \text{ m}$	$g_{2,y} = 0.4654 \text{ m}$	$g_{2,z} = -0.94 \text{ m}$
Attachment 3	$g_{3,x} = 0.4654 \text{ m}$	$g_{3,y} = 0.4654 \text{ m}$	$g_{3,z} = -0.94 \text{ m}$
Attachment 4	$g_{4,x} = 0.4654 \text{ m}$	$g_{4,y} = -0.4654 \text{ m}$	$g_{4,z} = -0.94 \text{ m}$

	Payloa	d Parameter						
Payload Mass (kg)	2000	Width (m)	1.88					
Length (m)	2.57	Height (m)	1.88					
Reference Area $(m^2)$	3.5344	Reference Length	1.88					
$I_{load}(kg * m^2)$	1178.1	1689.9	1689.9					
Cable Parameter								
Neutral Length (m)	50	Young's Modulus $(MPa)$	15000					
Cable diameter (mm)	6.4	Damping Constant (N-s/m)	5272.1					
Cable Angle, $\alpha_{g,i}$ (	deg)	33 to 39						
E	nvironn	nent Parameter						
Air density $(kg/m^3)$	1.225	Gravity $(m/s^2)$	9.81					
$W_{20} \ (m/s)$		15						
ConstantWind (m	n/s)	[(-2  to  -20), 0, 0]						

 Table A.3. Parameters for Monte Carlo: cable angle.

Table A.4. Parameters for Monte Carlo: cable stiffness.

Payload Parameter									
Payload Mass (kg)	2000	Width (m)	1.88						
Length $(m)$	2.57	Height (m)	1.88						
Reference Area $(m^2)$	3.5344	Reference Length	1.88						
$I_{load}(kg * m^2)$	1178.1	1689.9	1689.9						
Cable Parameter									
Neutral Length (m)	50	Cable diameter (mm)	6.4						
Young's Modulus (M	(Pa)	1e3 to $6e4$							
Cable Angle, $\alpha_{g,i}$ (deg)	37	Damping Constant (N-s/m)	vary						
Er	nvironm	ent Parameter							
Air density $(kg/m^3)$	1.225	Gravity $(m/s^2)$	9.81						
$W_{20} \ (m/s)$		15							
ConstantWind (m	(s)	[(-2  to  -20), 0, 0]							

	Payload Parameter									
Payload Mass (kg)	6500	Width (m)	1.88							
Length (m)	2.57	Height (m)	1.88							
Reference Area $(m^2)$	3.5344	Reference Length	1.88							
$I_{load}(kg * m^2)$	3828.9	5492.1	5492.1							
Cable Parameter										
Neutral Length (m)	50	Young's Modulus $(MPa)$	15000							
Cable diameter (mm)	12.8	Damping Constant (N-s/m)	19009							
Cable Angle, $\alpha_{g,i}$ (deg)	37									
Er	nvironm	ent Parameter								
Air density $(kg/m^3)$	1.225	Gravity $(m/s^2)$	9.81							
$W_{20} \ (m/s)$		15								
ConstantWind (m	/s)	[-4, 0, 0]								

 Table A.5. Parameters for transportation: four-helicopter.

 Table A.6. Attachment position for six-helicopter simulation.

	<b>1</b>	<b>1</b>	
Attachment 1	$g_{1,x} = -0.4654 \text{ m}$	$g_{1,y} = -0.4654 \text{ m}$	$g_{1,z} = -0.94 \text{ m}$
Attachment 2	$g_{2,x} = -0.4654 \text{ m}$	$g_{2,y} = 0.4654 \text{ m}$	$g_{2,z} = -0.94 \text{ m}$
Attachment 3	$g_{3,x} = 0.4654 \text{ m}$	$g_{3,y} = 0.4654 \text{ m}$	$g_{3,z} = -0.94 \text{ m}$
Attachment 4	$g_{4,x} = 0.4654 \text{ m}$	$g_{4,y} = -0.4654 \text{ m}$	$g_{4,z} = -0.94 \text{ m}$
Attachment 5	$g_{3,x} = 0 \text{ m}$	$g_{3,y} = 0.6582 \text{ m}$	$g_{3,z} = -0.94 \text{ m}$
Attachment 6	$g_{4,x} = 0 \mathrm{~m}$	$g_{4,y} = -0.6582 \text{ m}$	$g_{4,z} = -0.94 \text{ m}$

Payload Parameter					
Payload Mass (kg)	9500	Width (m)	1.88		
Length (m)	2.57	Height (m)	1.88		
Reference Area $(m^2)$	3.5344	Reference Length	1.88		
$I_{load}(kg * m^2)$	5596.1	8026.9	8026.9		
Cable Parameter					
Neutral Length (m)	65	Young's Modulus $(MPa)$	15000		
Cable diameter (mm)	12.8	Damping Constant (N-s/m)	20155		
Cable Angle, $\alpha_{g,i}$ (deg)	37				
Environment Parameter					
Air density $(kg/m^3)$	1.225	Gravity $(m/s^2)$	9.81		
$W_{20} \ (m/s)$		15			
ConstantWind (m/s)		[-4, 0, 0]			

 Table A.7. Parameters for transportation: six-helicopter.



# **Additional Simulation Results**

### B.1 Transportation: Four-Helicopter



Figure B.1. Wind velocity and payload air velocity: four-helicopter transport.



Figure B.2. Helicopter position errors: four-helicopter transport.



Figure B.3. Cable force on each helicopter: four-helicopter transport.
## B.2 Transportation: Six-Helicopter



Figure B.4. Wind velocity and payload air velocity: six-helicopter transport.



Figure B.5. Helicopter position errors: six-helicopter transport.



Figure B.6. Cable force on each helicopter: six-helicopter transport.

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