

# Random Geometric Graphs as a Model for Bounding the Endurance of Soaring Aircraft

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**Abstract**—Soaring aircraft and birds utilize the energy of atmospheric convection to remain aloft. As this source of energy gains attention as a method for prolonging the flight times of small UAVs, it is necessary to evaluate the conditions under which it is a viable option. The indefinite endurance problem for soaring aircraft is cast in terms of a random geometric graph. Tools developed for the analysis of random graphs are used to bound the relationship between aircraft efficiency and energy abundance in the atmosphere. A theoretical threshold is arrived at, over which indefinite endurance flights may be possible in an unbounded domain. Simulation results are presented that confirm the existence of a threshold which compares favorably to what is predicted by random graph theory. Extensions are made to missions restricted to finite domains as well.

## I. INTRODUCTION

Soaring aircraft often exclusively rely on the energy of atmospheric motion to sustain flight. With the right combination of pilot skill, knowledge about the surrounding airmass, and favorable atmospheric conditions, extraordinarily long flights are possible.

The aim of this work is to bound the characteristics of the environment, the aircraft, and the knowledge available that are required to sustain flight indefinitely using atmospheric energy alone. The approach taken is to represent the problem as a random geometric graph allowing us to relate theory and results developed for such graphs to address the limitations of a soaring system. Real-life scenarios are more complex than can be captured in the graph model, thus simulations are done with far fewer limiting assumptions. We proceed to show that the results are qualitatively the same and that the transition predicted by theory is evident in the simulation results.

One of many naturally occurring lift sources, thermal lift is caused by convective currents arising from solar heated hotspots on the ground[1]. When a mass of air rises faster than the sink rate of a bird or aircraft, it becomes usable as an energy source to sustain flight. Thermal lift is commonly used by soaring birds, manned gliders, and is gaining attention as a source of energy for small unmanned aircraft[2], [3], [4].

As convective lift cannot be explicitly resolved in regional-scale atmospheric models, thermal locations cannot be pre-

dicted with high resolution. This precludes the use of offline path planning tools to make use of the available energy. Long term environmental characteristics can be modeled in a probabilistic manner, so an exploration of the viability of sustained soaring flight in such an environment must be done in terms of probability as well.

This work assumes the use of thermal lift by aircraft as the sole source of energy to sustain flight. The developments that follow have the goal of enabling indefinite endurance by quantitatively stating the necessary conditions that enable it. Random graphs have been used as analogs to real-world conditions in similar environments where the statistical properties are known but the exact configuration is not[5]. An attempt is made to apply the conclusions to both endurance flights without boundaries and to loitering flight in a restricted region.

After an introduction to the environment and to soaring aircraft in Section II, the random geometric graph construction used in this work is presented in Section III. Results arrived at by applying theoretical developments to the indefinite endurance problem are presented for the case where the domain is unbounded in Section IV, and for the bounded case in Section V, then Section VI puts the results of simulations in perspective. Finally section Section VII concludes with the implications of this work on decision making for soaring aircraft.

## II. THE ENVIRONMENT AND THE AIRCRAFT

### A. The Convective Boundary Layer as a Source of Energy

The convective boundary layer (CBL) of the atmosphere extends from the ground to between one and four kilometers (the altitude  $z_i$ ), depending on the region and the time of year[6]. It is within this layer that soaring aircraft and birds typically operate, as it is where lift is most readily found. Thermals, the dominant feature in the CBL, are buoyant elements of the airmass that rise from a trigger point on the the ground to  $z_i$ . The lower 80% to 90% of a thermal[2] is typically of sufficient size, strength, and duration to allow aircraft to effectively climb within[1] (the top of the usable portion is denoted  $z_u$  here). Note that this altitude is consistent over a broad region.

Methods for predicting the prevalence of thermal lift in a given area on a particular day are available[7]. This allows for an expected lift source density  $\lambda$  to be approximated over a given region.

Whereas the locations of many atmospheric phenomenon can be predicted from terrain maps or meteorological forecasts, thermal lift is more difficult to precisely locate. For

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most of the following discussion, the assumption is made that the pilot has perfect knowledge of thermal locations. In cases where it is stated otherwise, it is assumed that a method capable of determining a thermal's existence and location with uniform probability  $p_d$  is available. Because thermals carry moisture from the surface, cumulus clouds often form at the top of a thermal column aiding their detection and justifying this assumption.

### B. Soaring Aircraft

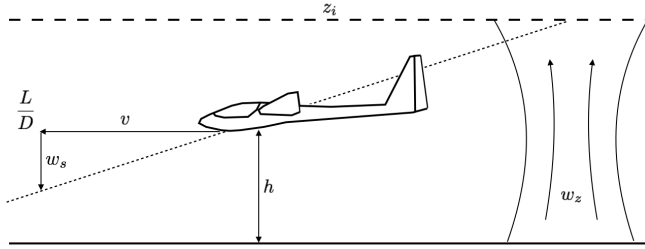


Fig. 1: The longitudinal kinematics of a soaring aircraft.

For aircraft that gain energy utilizing thermal lift, there are two primary phases of flight; soaring, in which the aircraft climbs by circling within the rising column of air, and gliding, where the altitude gained is transformed into distance. To capture the soaring phase of flight in this model it is sufficient to claim that an aircraft collocated with a thermal can climb to the altitude  $z_u$ . The efficiency with which an aircraft converts altitude to distance is expressed as a ratio of the aircraft's lift to drag,  $\frac{L}{D}$ , which is in turn a function of airspeed. A measure of how far an aircraft can travel for each unit of height lost (Figure 1), it is assumed here that a soaring aircraft will always fly at the airspeed that maximizes this ratio.

Modern sailplanes can have lift-to-drag ratios from 40-70. Birds are substantially less efficient, peaking around 10 for most soaring birds[8], and up to 15.3 for particularly aerodynamically efficient species[9]. In this work, the aircraft will be characterized solely by their glide efficiency  $\frac{L}{D}$ , as other parameters will have little effect on the problem formulation.

## III. PROBLEM FORMULATION

Consider a graph constructed from an infinite number of vertices that are connected pair-wise with probability  $p$ . An open cluster on this graph is a connected sets of vertices that can be accessed from one another. As  $p$  increases, the size of these connected clusters tends to increase as well.

Percolation on an infinite graph implies that there almost surely exists an open cluster that contains an infinite number of vertices. Identifying the conditions under which such a large connected set exists in a graph that represents an aircraft endurance problem will be the focus of this work.

**Definition 1.** Any random graph is said to percolate if there exists, with non-zero probability, an infinitely connected component somewhere on that graph.[10]

Percolation exhibits phase transition behavior at a critical probability,  $p_c$ . Graphs where  $p < p_c$  fail to percolate, implying that every cluster in the graph is of finite size (the subcritical case). When  $p > p_c$  the graph has an unbounded connected component with high probability (with probability 1)[10].

### A. Random Geometric Graphs

A random geometric graph is a random distribution of points that are not restricted to a lattice and may be connected pair-wise if conditions are satisfied. The most basic model of continuum percolation on a random geometric graph was introduced by Gilbert[11] as:

**Definition 2.**  $\mathcal{P}_\lambda$  is a Poisson process of intensity  $\lambda$  in  $\mathbb{R}^2$ , and  $r > 0$ . Connect any two points in  $\mathcal{P}_\lambda$  by an edge if the distance between them is less than  $r$ . The resulting graph is  $G_{r,\lambda}$ .

Gilbert's model was motivated to represent infinite networks of transceivers[11] and has since been popular in modeling various physical processes[12]. Recently, Gilbert's disk model has been used to support probabilistic completeness arguments in modern path planning strategies[13], as well as a model for bounding the speed at which a bird can fly through a forest[5].

**Definition 3.** The degree of a vertex in  $G_{r,\lambda}$  has a Poisson distribution with an expected value  $a = \lambda \pi r^2$ [10]

The value  $a$  is known as the expected degree of the graph or alternatively the connection area of  $G_{r,\lambda}$ .

As in the previous development, the graph  $G_{r,\lambda}$  percolates if there exists an infinite connected component somewhere on the graph. A value analogous to the critical probability  $p_c$ , the critical area  $a_c$  establishes percolation conditions on the continuous geometric graph[14]. For connection areas below the critical area  $a < a_c$ , every component of  $G_{r,\lambda}$  is certainly finite.

The exact value of this critical area is non-trivial to identify on a Gilbert-type graph. Gilbert's original paper[11] presented a geometric argument which placed the critical area in the broad range:

$$\frac{2\pi \ln 2}{3\sqrt{3}} \leq a_c \leq \frac{26\pi \ln 2}{3\sqrt{3}} \quad (1)$$

$$0.8382 \dots \leq a_c \leq 10.8960 \dots$$

The gap between these bounds means that they provide little insight when applied to practical engineering problems. Further analytical arguments allowed Hall[12] to narrow the bounds on the critical area for continuum percolation. More recently, studies relying on large scale Monte Carlo simulations[15] have put the range for the critical degree between 4.508 and 4.515.

### B. Comparison to the Parameters of a Random Graph

Comparisons to the Gilbert disk model are convenient because the model incorporates the distance between points

in  $\mathcal{P}_\lambda$  as a parameter and is thus easily applied to physical processes. Here we consider an idealized scenario in which a soaring aircraft is not bounded physically and has perfect knowledge of atmospheric conditions. In this world, an aircraft is free to choose any lift source within reach, climb in it, and transit to another.

Two thermals will be considered connected if from one, after climbing through the working altitude of that lift source, an aircraft can reach the other. This makes the connection of two lift sources a function of the usable portion of the convective boundary layer  $z_u$  as well as the aircraft's aerodynamic efficiency  $\frac{L}{D}$ :

$$r = z_u \left( \frac{L}{D} \right) \quad (2)$$

Thermal trigger locations, being difficult to predict, will be modeled as uniformly randomly distributed over the domain of interest. Over homogeneous and flat terrain, this assumption is approximately valid[16]. Lift source locations in the two dimensional plane are associated with the points of the Poisson process  $\mathcal{P}_\lambda$ . The density parameter  $\lambda$  can then be chosen to approximate the expected prevalence of lift.

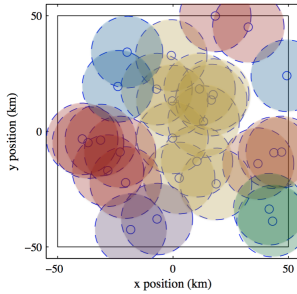


Fig. 2: Schematic showing thermal lift source locations as small blue circles distributed randomly in a 100x100 km region. The accessible region (for an aircraft with an  $\frac{L}{D}$  of 15,  $z_u = 1\text{km}$ ) from the top of each thermal is indicated by the large dashed circles.

Figure 2 shows an example environment cast as a random graph. Several clusters of connected lift sources are colored. An aircraft located in a lift source that is a part of a colored cluster is able to reach any other location colored similarly. That aircraft cannot jump to a cluster of a different color. However, an aircraft with a larger  $\frac{L}{D}$  will see a more connected graph, and an efficient enough aircraft will face only a single cluster. Similarly, if the thermal locations were more densely distributed, an aircraft with the same performance could likely access a broader region.

### C. Similarity Observation

On comparing a soaring aircraft problem to a random graph, the first meaningful conclusion can be taken from the fact that the structure of the graph  $G_{r,\lambda}$  depends only on the expected degree  $a$ . In other words, the graph will exhibit the same behavior, regardless of the individual values of  $r$  and  $\lambda$ , provided the connection area  $a = \lambda \pi r^2$  remains

constant[10]. Note that this concept does not require that a graph percolates.

Using the expression for  $r$  given in Equation 2, an aircraft or bird operating under the restrictions of the graph  $G_{r_0,\lambda_0}$  will have the same performance as one restricted to  $G_{r_1,\lambda_1}$ , provided the relation holds:

$$\lambda_0 z_{u,0}^2 \left( \frac{L_0}{D_0} \right)^2 = \lambda_1 z_{u,1}^2 \left( \frac{L_1}{D_1} \right)^2 \quad (3)$$

The value of knowledge about the environment can be quantified using this relation. If thermals can be detected with uniform probability, then the relation becomes:

$$\lambda_0 z_{u,0}^2 p_{d,0} \left( \frac{L_0}{D_0} \right)^2 = \lambda_1 z_{u,1}^2 p_{d,1} \left( \frac{L_1}{D_1} \right)^2 \quad (4)$$

where  $p_{d,\cdot}$  is the uniform detection probability.

## IV. INDEFINITE ENDURANCE IN A BOUNDLESS DOMAIN

Percolation in the scenario presented in Section III-B implies that there is a positive probability that an infinite number of lift sources of sufficient strength to prolong the length of a flight are accessible to the aircraft. For connection areas above the critical area  $a_c$ , there is an infinite component to  $G_{r,\lambda}$ , and thus an aircraft should be able to remain aloft indefinitely if it is located within this infinite component. If the graph  $G_{r,\lambda}$  fails to percolate, it implies that an aircraft will eventually be unable to reach a source of lift and will be forced to land.

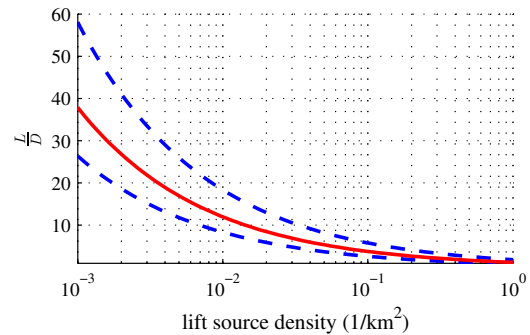
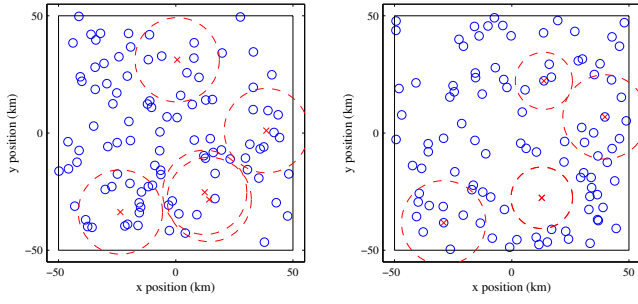


Fig. 3: Selected percolation bounds dictated by bounds on critical degree. The blue lines show the best theoretical bounds given by Hall[12]. The solid red line shows the best approximation arrived at through numerical simulation[15].

For soaring flight indefinite endurance is possible if the graph  $G_{r,\lambda}$  percolates, i.e. if:

$$\lambda \pi z_u^2 \left( \frac{L}{D} \right)^2 \geq a_c \quad (5)$$

For the bounds on critical degree  $a_c$  defined in Section III-A, the aircraft performance required for indefinite endurance in a thermal field of given density is plotted in Figure 3.



(a) Simulation at  $t = 0$ . (b) Simulation at  $t = 150$  minutes.

Fig. 4: An example simulation of 5 aircraft ( $\frac{L}{D} = 18$ ,  $z_u = 1\text{km}$ ) in a  $100 \times 100$  km region with a lift source density of  $0.01$  thermals per  $\text{km}^2$ . The accessible region for each aircraft is plotted with a red circle. Thermal locations are indicated by blue circles.

### A. Simulation

While graph theory provides significant insight into the relationship between atmospheric conditions and required aircraft performance, a key assumption was required, namely that a given thermal persists indefinitely. With Monte Carlo simulations this assumption can be lifted. The purpose of the simulations is to show that the same transition threshold exists when the problem is modeled with more fidelity and to demonstrate how well established theory can predict the threshold.

A thermal dominated environment model developed at NASA Dryden[17] from measured data is used with parameters  $z_i = 1401\text{m}$  and  $w^* = 2.56\text{m/s}$ . These values describe a mean thermal where  $w_z$  varies with altitude, reaching a maximum strength of  $2.74\text{m/s}$  at  $\frac{h}{z_i} = 0.21$  with the usable portion extending to  $z_u \approx 1100\text{m}$ . An individual thermal's strength is drawn from a normal distribution about the mean with  $\sigma = 20\%$ . Atmospheric sink  $w_d$  is modeled in the spaces between thermals to enforce mass conservation.

Thermal lifespans have an expected value of 20 minutes; on failure another is generated at a new location to keep the density of lift sources constant. Trigger locations are uniformly randomly distributed over the entire simulated region but a separation of no less than  $0.3z_i$  is imposed between any two thermals to better model the physical process of atmospheric convection[18].

Aircraft motion is simulated with a simple kinematic model, the longitudinal component of which is shown in Figure 1. Each is allowed to transit the plane in search of lift at speed  $v$ . The sink rate of all aircraft is  $0.5\text{m/s}$  while  $v$  is varied with  $\frac{L}{D}$ . When collocated with a lift source an aircraft is modeled to climb at  $0.75w_z - w_s$  to simulate imperfect utilization of the full strength of a thermal. Otherwise an aircraft sinks at the rate  $w_s + w_d$ . If an aircraft's altitude reaches 0 at any point it is removed from the simulation.

Aircraft seek to stay aloft as long as possible, thus their strategy is simple: transit to the nearest detected thermal and

remain within the lift while it exists. Two snapshots displaying the dynamics of a simulation are shown in Figure 4a and Figure 4b.

Each data point reflects 10 trials of 1000 independently acting aircraft in a  $4000 \text{ km}^2$  environment. Each aircraft is placed at a random location in the middle of the domain to avoid edge effects. Missions last 12 hours, a period over which it is reasonable to expect soaring weather to last in the summer[2]. The performance parameter  $\frac{L}{D}$  and the environmental parameter  $\lambda$  are varied in a band around the expected transition region (Equation 5) to identify the behavior of the system in an area that the theory in Section III predicts will be meaningful.

### B. Simulation Results

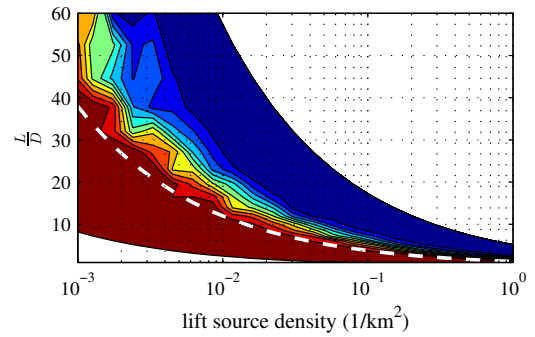


Fig. 5: Colored contours correspond to the percentage of 10,000 aircraft that survive for 12 hours in an environment where  $z_u \approx 1100\text{m}$ . Blue implies 90-100% of aircraft survive, red implies 0-10% of aircraft survive. The dotted line shows the theoretical threshold  $a_c \approx 4.51$ .

The results of these simulations are plotted in Figure 5. The threshold given by the critical area  $a_c \approx 4.51$  is plotted in relation to the survival statistics of the simulated aircraft. Clearly shown is a threshold below which long endurance is not possible. The trend in average aircraft altitude over time for a moderate lift source density is plotted in Figure 6.

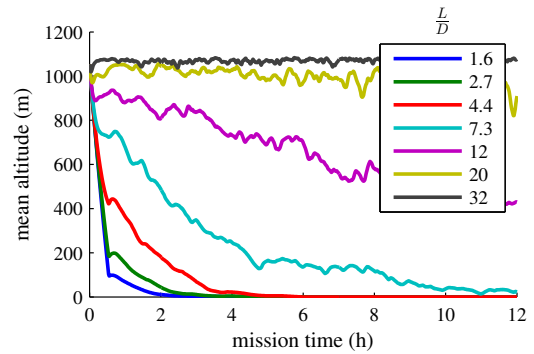


Fig. 6: Trend in mean aircraft altitude plotted versus mission time for an expected lift source density fixed at  $0.01$  per square kilometer with increasing aircraft performance. Note that the maximum attainable altitude  $z_u \approx 1100\text{m}$ .

## V. INDEFINITE ENDURANCE IN A FINITE DOMAIN

The conditions derived in the previous sections are not immediately applicable to aircraft deployment. It is difficult to imagine a scenario in which the domain of interest is not geographically restricted in some way; either by operational requirements or by bounds mandated by environmental parameters. Perhaps a more reasonable representation arises when the random graph is confined to a finite domain.

Percolation on a graph with a finite number of vertices (such as one restricted to a finite domain) is defined as the point at which the largest connected component in the graph contains at least a constant (non-zero) fraction of the total number of vertices in the graph[14]. The definition of a disk graph on the finite planar domain is as follows.

**Definition 4.**  $\mathcal{P}_\lambda$  is a Poisson process of intensity  $\lambda$  in  $[0, \ell]^2$ , and  $r > 0$ . Connect any two points in  $\mathcal{P}_\lambda$  by an edge if the distance between them is less than  $r$ . The resulting graph is  $G_{r,n}$ .

On  $G_{r,n}$ ,  $n = \lambda\ell^2$  is the expected number of vertices in the finite region. The expected degree of a point in  $G_{r,n}$  is  $d(n) = \pi \left(\frac{r}{\ell}\right)^2 n$ . The percolation threshold in the restricted graph[14] in terms of the parameters of this problem is:

$$\lim_{n \rightarrow \infty} \frac{L_1(G_{r,n})}{n} = 0 : r < \sqrt{\frac{a_c}{\pi\lambda}}$$

$$\lim_{n \rightarrow \infty} \frac{L_1(G_{r,n})}{n} > 0 : r > \sqrt{\frac{a_c}{\pi\lambda}} \quad (6)$$

where  $a_c$  is the critical degree in an infinite domain presented previously, and  $L_1(G_{r,n})$  is the number of vertices in the largest connected component of  $G_{r,n}$ .

The transition threshold over which percolation occurs in a finite region is the same as that in an infinite domain (Equation 5), and provides a similarly conservative lower bound on aircraft performance parameters.

### A. The Environment as a Connected Graph

To further examine the graph  $G_{r,n}$  restricted to the finite domain, the conditions under which the graph becomes connected can be determined. These conditions lead to stronger bounds on aircraft performance, as a connected graph requires that there is a path from any lift source to any other. Connectivity of the graph is a stricter requirement than percolation, thus if  $G_{r,n}$  is connected, it also percolates[13].

A  $k$ -connected graph will not become disconnected by the removal of  $k - 1$  or fewer vertices[14]. The critical degree to ensure that the graph is  $k$ -connected is identified by[10]:

$$P(G_{r,n} \text{ is } k\text{-connected}) \rightarrow e^{-e^{(k-1) \log \log n + \log n - d(n)}} \quad (7)$$

In particular:

$$\lim_{n \rightarrow \infty} P(G_{r,n} \text{ is } k\text{-connected}) \rightarrow 0 : r < f(s, \lambda, \ell) \quad (8)$$

$$\lim_{n \rightarrow \infty} P(G_{r,n} \text{ is } k\text{-connected}) \rightarrow 1 : r > f(s, \lambda, \ell) \quad (9)$$

where:

$$f(s, \lambda, \ell) = \sqrt{\frac{\log(\lambda\ell^2) + (k-1) \log \log(\lambda\ell^2)}{\pi\lambda}} \quad (10)$$

The implication of this as applied to our problem is that an aircraft can afford to miss (either bypass or fail to detect) any  $k - 1$  lift sources and still be certain that it can reach any other lift source in the finite domain.

### B. An Isolated Lift Source

In the subcritical case, the graph  $G_{r,n}$  is not connected, implying the existence of isolated vertices in the domain  $[0, \ell]^2$ . Clearly an aircraft located in a lift source from which it cannot reach any other lift is contrary to the goal of indefinite endurance, thus this situation should be examined.

The fraction of the total number of vertices that are expected to be isolated is of immediate interest. This has implications in predicting the failure rate of aircraft. Defining  $X_0$  as the number of isolated vertices in the graph  $G_{r,n}$ , the ratio[10] reduces to an exponential random variable independent of the area of the region:

$$\frac{\mathbb{E}(X_0)}{n} = e^{-(\pi r^2 \lambda)} \quad (11)$$

If an environment is to support an aircraft aloft for long periods, the number of isolated thermals should be minimal and the probability of ending up in one small.

### C. Finite Domain Simulation

Simulation of aircraft endurance missions in a finite region were conducted with similar parameters to those in the larger region. Two square regions of 100km<sup>2</sup> ( $\ell = 10,000\text{m}$ ) and 2,500km<sup>2</sup> ( $\ell = 50,000\text{m}$ ) were examined. Fifty simulations of 100 independent aircraft were conducted for each pair of lift source density and aircraft performance parameters.

### D. Simulation Results

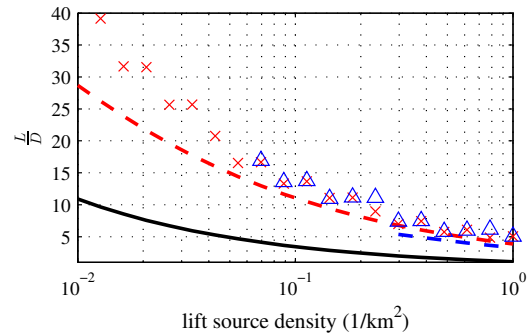


Fig. 7: Indicated by red x's are the values of  $\frac{\ell}{D}$  over which 99% of aircraft survive for 12 hours of simulation in a domain where  $\ell = 50\text{km}$  and  $z_u \approx 1100\text{m}$ . The dashed red line shows the threshold over which the same domain is 25-connected. Blue triangles and the blue line represent the same for a domain where  $\ell = 10\text{km}$ . The percolation threshold for the finite domain is plotted in black.

The relations in Figure 7 indicate that some level of connectedness is a better lower bound on aircraft performance than is percolation. Plotted is the point, for each environmental parameter, over which 99% of simulated aircraft survive. Also plotted in Figure 7 are the lines indicating the threshold for  $k = 25$  in each domain size.

## VI. DISCUSSION

Casting the problem of indefinite aircraft endurance into the framework of a random geometric graph allows for a powerful set of tools to be applied to the problem. The validity of the comparison was borne out in the simulation results plotted in Figure 5. A threshold is evident below which nearly every simulation results in the loss of all aircraft, indicating that all subgraphs are small enough that an aircraft will eventually run out of reachable lift sources. As the expected degree increases above the threshold, the percentage of aircraft that survive the duration of the simulation increases (Figure 6).

The structure of the graph depends only on the expected degree as given in Equation 4. This allows graphs to be compared and therefore the parameters that enable prolonged flight in different environments or by different vehicles. A classic observation of soaring pilots is that birds are able to stay aloft on days where a glider cannot find lift[8]. These soaring birds, despite being aerodynamically less efficient, are natural aviators. They are more likely to detect lift sources in their environment and they fly at a lower wing loading, meaning they can take advantage of tighter (and more abundant[18]) thermals than can a human piloted glider. The net effect is that the expected degree of the graph faced by the birds is greater than that of envious pilots.

The concept of percolation as applied to soaring flight has important implications. The relationship between lift source density and aircraft performance plotted in Figure 3 shows a threshold over which indefinite endurance should be possible. The threshold location identified through simulation (Figure 5) lies very near to the theoretical transition, but shows that percolation is a conservative measure.

Whereas percolation only guarantees that a large connected component exists somewhere on the graph, the degree to which the graph is connected leaves room for an aircraft to miss lift sources and still travel to another before landing out. When attempting to bound the performance of an aircraft such that long endurance missions are possible, connectedness likely is a more relevant measure. Simulations in finite domains show that  $k \approx 25$  provides a good lower bound for long endurance times (Figure 7).

## VII. CONCLUSIONS

Presented in this work is a random graph representation of the problem of extended endurance at low altitudes for soaring aircraft. Tools from the theory of random graphs were applied to the problem to yield a relationship between aircraft performance and the density of lift sources in the environment. In the sub-critical case, comparisons were made that quantify the utility of knowledge, additional

performance, and more abundant lift. In the critical case, a theoretical threshold was identified through theory and confirmed through simulation that specifies the relationship between performance and the environment that allow indefinite endurance. The results of this work can quantify the risk to a mission posed by predicted atmospheric conditions and inform decision making strategies if soaring flight is to be utilized to extend endurance.

## VIII. ACKNOWLEDGMENTS

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